Incentive Compatibility in Experiments with Incomplete Preferences

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October 29, 2026

Abstract

Experiments that elicit preferences usually presume completeness, one of the most contested axioms of choice theory. I study incentive compatibility in experiments that let subjects report incomplete preferences. To tell incomparability apart from indifference, I attach a sure monetary payment to each option and require incomparability to survive small changes in that payment. The frontier of incentive compatibility under incomplete preferences is comparable-side strict incentive compatibility: subjects have a strict incentive to report truthfully when they can compare two options, and only a weak incentive when they cannot. Reported incomparability is therefore a conservative measure that never overstates incompleteness. I construct a class of mechanisms that attain this frontier and apply to any domain. In a laboratory experiment, these mechanisms also prove behaviourally incentive compatible.

Keywords: Incomplete preference; incentive compatibility; experimental design; revealed preference; mechanism design.

1 Introduction

Elicitation of preferences is a common practice in economic experiments. Almost every experiment elicit preferences by forcing subjects to rank the options. An implicit assumption behind this forced method is complete preference, which means that decision makers can always compare any two options, either $x \succsim y$ or $y \succsim x$. A growing body of experimental evidence documents violations of completeness: across studies, the share of subjects whose choices point to incomplete preferences ranges from about one fifth to nearly three quarters

(Cettolin & Riedl, 2019; Gerasimou, 2025; Halevy, Walker-Jones, & Zrill, 2025; Nielsen & Rigotti, 2022). Ignoring incompleteness can also lead experimenters to misinterpret the behavior they observe. Gao and Halevy (2026) find that incomplete preferences are associated with status quo retention, which suggests that incompleteness is a source of status quo bias that complements the standard explanation based on loss aversion.

These findings call for experiments that do not force subjects to rank every pair of options and instead allow them to report incomparability directly. Such designs must also be incentive compatible: truth-telling must yield an optimal outcome, and lies must be penalized. Experiment designs that violate incentive compatibility can distort what is measured. For instance, truth-telling is a weakly dominant strategy in the second-price auction, and the Becker–DeGroot–Marschak (BDM) mechanism applies this logic to individual valuation, with the multiple price list (MPL) as its discrete equivalent. However, in the domain of risk, neither mechanism is incentive compatible when preferences violate independence (Karni & Safra, 1987), and Freeman, Halevy, and Kneeland (2019) find that subjects choose a risky lottery over a sure payment significantly more often when the choice is embedded in a list, which makes them appear less risk averse. This paper studies how to elicit preferences in an incentive-compatible way when they may be incomplete.

Specifically, I address three questions. First, which notions of incentive compatibility are attainable when preferences may be incomplete? The answer traces a frontier of incentive compatibility. Second, which mechanisms attain this frontier? Third, how do these mechanisms perform in the laboratory?

To answer the first question, I study experiments that use a random incentive system: the subject answers every question, and a random device then determines her payment from her reports. Random incentive systems are the most common payment scheme in experiments, and they are tractable: each report induces an act that pays an outcome in each state of the device, so incentive compatibility can be analyzed state by state, following Azrieli, Chambers, and Healy (2018). The standard definition of incentive compatibility does not carry over directly to incomplete preferences, because the outcome of truth-telling may be incomparable to the outcome of a lie. Therefore, I start from a weak incentive compatibility, which requires only that no lie yield a strictly better outcome than the truth, and ask which lies can be penalized on top of this weak incentive compatibility. Incomplete preferences give rise to two kinds of lies. A *comparable-side lie* misreports a pair of options that the subject can rank, either by reversing her ranking or by claiming that she cannot rank them; an *incomparable-side lie* ranks a pair that she cannot compare.

The two kinds of lies differ in whether they can be penalized. No mechanism can penalize incomparable-side lies, whereas comparable-side lies can be penalized, provided that incomparability is not too thin. In a finite experiment, it is impossible to distinguish an indifferent subject who claims incomparability from one whose incomparability disappears after an arbitrarily small change in money. Experimenters must therefore choose a threshold b and distinguish indifference from incomparability that persists under monetary changes smaller than b; in other words, incomparability must be thick enough to be separated from indifference in a finite experiment. The resulting notion, which I call b-robust comparable-side incentive compatibility (b-RCSIC), characterizes the frontier of incentive compatibility under incomplete preferences.

Turning to the second question, the frontier can be attained within random incentive systems. However, random incentive systems raise concerns in the domains of risk and ambiguity. As noted above, they are incentive compatible under risk only if preferences satisfy independence, and under ambiguity the random device gives subjects an opportunity to hedge (Baillon, Halevy, & Li, 2022). I therefore look for mechanisms that attain the frontier without a random device. I start from the Accumulative Best Choice (ABC) mechanism of Y. Li (2021). In ABC, options are revealed one at a time: the subject holds a current option, compares it in each round with the newly revealed option, and keeps the one she prefers, and her payment is the option she holds at the end. Since the payment is simply the final holding, no random device is involved, and Y. Li (2021) shows that ABC is incentive compatible for every complete and transitive preference.

ABC is designed for complete preferences, and I extend it in two ways. First, the subject may report that she cannot rank the two options; the mechanism must then decide which option she keeps for the next round, and to preserve incentive compatibility it must never leave her holding an option worse than one she has already seen. Second, the subject works through several lists rather than one, without knowing how many will follow, and her earlier answers bind if a comparison reappears. The second extension is essential: with a single list, every such mechanism leaves some comparable-side lies unpenalized, whereas with multiple lists these lies can be penalized. I call the resulting mechanisms Multi-list Accumulative Maximal Choice (AMC) mechanisms. Some of them attain the frontier without any random device and therefore can be applied to different domains.

I take Multi-list AMC mechanisms to the laboratory to test whether they are also behaviourally incentive compatible. In an experiment with 422 subjects, each subject completes seven lists of risky and ambiguous prospects under one of three mechanisms: two Multi-list AMC mechanisms, which allow subjects to report that they cannot rank two options, and

a Multi-list ABC mechanism, which forces a ranking, as in standard designs. Subjects in a verification arm then complete one additional round, in which a previously answered comparison is presented again as a single choice and the chosen option determines their payment. This round is equivalent to a one-task experiment which is a common benchmark for testing incentive compatibility.

Three findings support the behavioral incentive compatibility of AMC. First, subjects answer correctly when the truthful answer is known. Under AMC, 91% of subjects report a tie when an option is compared with an identical copy, and only 1.4% report that they cannot rank the two; under both AMC and ABC, subjects rank a dominating option above a dominated one in about 98% of comparisons. Second, and most importantly, AMC elicits more reliable rankings than ABC: the verification choice confirms 83.3% of the rankings reported under AMC, compared with 71.4% under ABC ($p = 0.024$). Third, reports of incomparability are associated with greater decision difficulty, as one would expect if they reflect a genuine inability to compare. Subjects take about 65% longer on comparisons they report they cannot rank (28.8 versus 17.4 seconds), and they are four times as likely to describe these comparisons as difficult in the exit survey (21.7% versus 5.3%). Overall, 21.6% of subjects under AMC report incomparability at least once, most often in comparisons involving ambiguity.

Section 2 reviews the related literature. Section 3 sets up the model. Section 4 characterizes the frontier of incentive compatibility with incomplete preferences. Section 5 put forward a class of mechanisms that can attain the frontier. Section 6 describes the experiment design, Section 7 reports the results, and Section 8 concludes.

2 Literature Review

This paper are related to 3 branches of literature.

The first studies incentive compatibility in experiments (e.g., Azrieli et al., 2018; Azrieli, Chambers, & Healy, 2020; Baillon et al., 2022; Danz, Vesterlund, & Wilson, 2022; Freeman et al., 2019; Karni & Safra, 1987; S. Li, 2017). These papers assume, explicitly or implicitly, that preferences are complete. I build on the framework of Azrieli et al. (2018) and extend the analysis of incentive compatibility to incomplete preferences, identifying which notions of incentive compatibility remain attainable.

Secondly, there are experimental works on the identification of incomplete preferences. Among them, some experiment designs are tailored for a specific domain. For example,

Cettolin and Riedl (2019) Halevy et al. (2025) study incomplete preferences in the domain of ambiguity. Agranov and Ortoleva (2025) study deliberate randomization and conjecture that incompleteness may be related to it. Our paper aims to put forward a experiment design that can be applied to different domains. Some experimental works uses choices deferral as a behavior signature to identify incomplete preferences (e.g., Costa-Gomes, Cueva, Gerasimou, & Tejiščák, 2022; Danan & Ziegelmeyer, 2006; Gerasimou, 2025). This approach presumes that a DM who cannot rank two options chooses to defer, and deferral admits other explanations, such as a preference for flexibility or a desire to learn about one's own preferences. My design instead lets the DM report incomparability directly and makes no assumption about how incomparability manifests in choice. Several existing elicitation methods for incomplete preferences are, moreover, not incentive compatible in theory. Nielsen and Rigotti (2022) elicit incomplete preferences over monetary gambles in a non-forced choice environment, in which subjects may report their incomplete preferences directly and an algorithm determines their payment; the mechanism is not incentive compatible in theory but is argued to be so behaviourally. **?** use a modified multiple price list to elicit both imprecision intervals and certainty equivalents for a set of lotteries, but the imprecision intervals are not incentivized, and the elicitation procedures in **?** and **?** are unincentivized as well. This paper proposes mechanisms that are incentive compatible in theory and tests their behavioural incentive compatibility in the laboratory.

Thirdly, this paper is also related to theoretical works on incomplete preference, (e.g., Aumann, 1962; Dubra, Maccheroni, & Ok, 2004; Faro, 2015; Galaabaatar & Karni, 2013; Ghirardato, Maccheroni, & Marinacci, 2004; Gilboa, Maccheroni, Marinacci, & Schmeidler, 2010; Nau, 2006; Ok, 2002; Ok, Ortoleva, & Riella, 2012; Riella, 2015). In particular, Eliaz and Ok (2006) put forward a theoretical approach to distinct incomplete preferences and indifference from choices. They do not provide a clear pathway to experiment design from this theoretical approach. We use an assumption different but related and we put forward a pathway to experiment design. Ok et al. (2012) separates indecisiveness into two sources: imprecise beliefs (modeled via multiple priors) and imprecise tastes (modeled via multiple utility functions). In the experiment, I use both risky and ambiguous prospects to compare the two sources empirically.

3 Model

3.1 Primitives

Let X denote the set of objects of interest. I impose no structure on X: its elements may be risky lotteries, ambiguous prospects, or alternatives from any other domain. The decision maker (DM)'s preference is a binary relation $\succsim$ on X, which the experimenter seeks to elicit. As usual, $\succ$ and $\sim$ denote the asymmetric and symmetric parts of $\succsim$, respectively; that is, $x \succ y$ if $x \succsim y$ and not $y \succsim x$, and $x \sim y$ if $x \succsim y$ and $y \succsim x$.

Throughout the paper, I assume that $\succsim$ is a preorder, that is, it satisfies the following two assumptions.

Assumption 1 (Transitivity)**.** *For all $x, y, z \in X$, if $x \succsim y$ and $y \succsim z$, then $x \succsim z$.*

Assumption 2 (Reflexivity)**.** *For all $x \in X$, $x \succsim x$.*

Notably, I do not require $\succsim$ to be complete. Two objects $x, y \in X$ are *incomparable*, written $x \bowtie y$, if neither $x \succsim y$ nor $y \succsim x$. Hence, for any $x, y \in X$, exactly one of $x \succ y$, $y \succ x$, $x \sim y$, and $x \bowtie y$ holds.

3.2 Domain Expansion

On X alone, Assumptions 1 and 2 do not guarantee that the experimenter can distinguish indifference from incomparability. To separate the two, I augment each object with a sure monetary payment. The expanded domain is $X \times \mathbb{R}$, whose typical element (x, η) consists of an object $x \in X$ and a sure amount of money $\eta \in \mathbb{R}$. Let $\succsim^+$ denote the decision maker's preference over $X \times \mathbb{R}$, and define its asymmetric part $\succ^+$, symmetric part $\sim^+$, and incomparability relation $\bowtie^+$ analogously to $\succ$, $\sim$, and $\bowtie$. I impose the following assumptions on $\succsim^+$.

Assumption 3 (Transitivity)**.** *For all $(x, \eta), (y, \gamma), (z, \zeta) \in X \times \mathbb{R}$, if $(x, \eta) \succsim^+ (y, \gamma)$ and $(y, \gamma) \succsim^+ (z, \zeta)$, then $(x, \eta) \succsim^+ (z, \zeta)$.*

Assumption 4 (Reflexivity)**.** *For all $(x, \eta) \in X \times \mathbb{R}$, $(x, \eta) \succsim^+ (x, \eta)$.*

Assumption 5 (Agreement on X)**.** *For all $x, y \in X$, $x \succsim y$ if and only if $(x, 0) \succsim^+ (y, 0)$.*

Assumption 6 (Monotonicity in Money)**.** *For all $(x, \eta), (x, \eta') \in X \times \mathbb{R}$, if $\eta > \eta'$, then $(x, \eta) \succ^+ (x, \eta')$.*

Assumption 7 (Thick Incomparability)**.** *For every $(x, \eta) \bowtie^+ (y, \gamma)$, the sets $\{\eta' \in \mathbb{R} :$*

$(x, \eta') \bowtie^+ (y, \gamma)\}$ *and* $\{\gamma' \in \mathbb{R} : (x, \eta) \bowtie^+ (y, \gamma')\}$ *are open in* $\mathbb{R}$.

I call $\succsim^+$ *admissible* if it satisfies Assumptions 3–7, and I write $\mathcal{R}$ for the set of admissible relations.[1]

Assumption 7 requires incomparability to be robust to small changes in money: if $(x, \eta) \bowtie^+ (y, \gamma)$, then the two pairs remain incomparable after a sufficiently small change in either η or γ. The assumption is implied by the following continuity-in-money condition: for all $x \in X$ and $(y, \gamma) \in X \times \mathbb{R}$, the sets $\{\eta \in \mathbb{R} : (x, \eta) \succsim^+ (y, \gamma)\}$ and $\{\eta \in \mathbb{R} : (y, \gamma) \succsim^+ (x, \eta)\}$ are closed in $\mathbb{R}$. Indeed, the union of these two sets is the set of η at which (x, η) is comparable to (y, γ). This union is closed, so its complement $\{\eta \in \mathbb{R} : (x, \eta) \bowtie^+ (y, \gamma)\}$ is open. Exchanging the roles of the two objects yields the openness of the second set in Assumption 7. The converse, however, does not hold. Because Assumption 7 constrains only incomparable pairs, it holds vacuously when $\succsim^+$ is complete, yet a complete $\succsim^+$ can violate continuity in money: one of the two sets in the condition may fail to be closed. Assumption 7 is therefore weaker than continuity in money.

Assumption 7 is key to differentiating indifference and incomparability. Intuitively, thick incomparability means that a marginal monetary improvement or deterioration will not break incomparability, so it is robust to small perturbations in the monetary component. This stands in contrast to indifference, which can be broken by any strict improvement or deterioration. Eliaz and Ok (2006) uses a different but related assumption, regularity, to distinguish indifference and incomparability. A preference relation on a nonempty set X is said to be *regular* if, for any $x, y \in X$ with $x \bowtie y$, there is a $z \in X$ such that either $x \bowtie z$ and y and z are strictly ordered by $\succ$, or $y \bowtie z$ and x and z are strictly ordered by $\succ$. However, the finite discrete X in any experiment may lack the necessary witness z. By extending X to $X \times \mathbb{R}$ together with the assumption of thick incomparability, such a witness always exists: $\forall x \bowtie y$, $\exists\, \gamma > 0$ such that $(x, 0) \bowtie^+ (y, \gamma)$ and $(y, \gamma) \succ^+ (y, 0)$.

3.3 Experiment and Mechanism

I focus on direct elicitation of preference. An experiment consists of a finite list of binary decision problems $D = (D_1, \ldots, D_k)$, where $D_i = \{d_{1i}, d_{2i}\}$ [2]. For each D_i, the set of messages is

$$M_i = \left\{ d_{1i} \hat{\succsim} d_{2i},\ d_{2i} \hat{\succsim} d_{1i},\ d_{2i} \hat{\bowtie} d_{1i} \right\},$$

[1] Given Assumption 5, Assumptions 1 and 2 follow from Assumptions 3 and 4. For instance, if $x \succsim y$ and $y \succsim z$, then $(x, 0) \succsim^+ (y, 0) \succsim^+ (z, 0)$, so $x \succsim z$.

[2] I set each question to be binary because the message structure will be complicated for incomplete preferences if there are more than two choices in a menu.

The overall message space is $M = \prod_i M_i$. A typical element in M is denoted as m. The first two messages combine strict preferences and indifferences together. There is no separate message for indifference and strict preferences as a DM who are indifferent between two options can not be penalized by reporting a strict preferences even when with the assumption of completeness (Lemma A.4 in the Appendix).

Truthful reports. Given a true preference $\succsim^+$, the set of truthful reports in D_i is

$$T_i(\succsim^+) = \begin{cases} \{d_{1i} \hat{\succsim} d_{2i}\} & \text{if } d_{1i} \succ^+ d_{2i}, \\ \{d_{2i} \hat{\succsim} d_{1i}\} & \text{if } d_{2i} \succ^+ d_{1i}, \\ \{d_{1i} \hat{\succsim} d_{2i},\ d_{2i} \hat{\succsim} d_{1i}\} & \text{if } d_{1i} \sim^+ d_{2i}, \\ \{d_{1i} \hat{\bowtie} d_{2i}\} & \text{if } d_{1i} \bowtie^+ d_{2i}, \end{cases}$$

and $T(\succsim^+) = \prod_i T_i(\succsim^+)$ is the set of truthful message profiles.[3]

Recall that the experimenter's primary interest is in eliciting $\succsim$ over X, so not all decision problems in D are equally important. The set of *core tasks* is

$$D_{core} = \{D_i \in D :\ d_{1i} = (x, 0) \text{ and } d_{2i} = (y, 0) \text{ for some } x, y \in X\},$$

the decision problems in which both options carry a zero monetary component. By Assumption 5, the DM's ranking in a core task coincides with her ranking of x and y under $\succsim$. The remaining tasks serve as auxiliary instruments, so the strict incentive requirements in Section 4 apply only to lies on core tasks. A message profile m' *lies* on a core task D_i if $m'_i \notin T_i(\succsim^+)$. It makes a *comparable-side lie* if it lies on some core task whose options are comparable under $\succsim^+$, and an *incomparable-side lie* if it lies on some core task D_i with $d_{1i} \bowtie^+ d_{2i}$.

To trace the frontier of incentive compatibility under incomplete preferences, I build on the framework of Azrieli et al. (2018) and focus on experiments that use a Random Incentive System (RIS): the DM answers every decision problem in D, and a random device then determines her payment. The leading example, which Azrieli et al. (2018) study, is the Random Problem Selection (RPS) mechanism, in which the device draws one decision problem and the DM receives the option she chose in it. I allow both the device and the payment rule to be more general. For example, the device may draw a subset of options rather than a single decision problem, and the mechanism may then pay one option from that subset

[3] Azrieli et al. (2018) assume strict preferences. I allow indifference because distinguishing it from incomparability is central once preferences may be incomplete.

based on the DM's reports. Whatever the device and the payment rule, once the reports are fixed, the payment depends only on the realization of the device.

Let Ω denote the set of realizations, or *states*, with typical element ω. Each message profile m therefore induces an *act*, a map from Ω to $X \times \mathbb{R}$, and I write f, g, and h for typical acts in $(X \times \mathbb{R})^{\Omega}$. A *mechanism* is a map $\phi : M \to (X \times \mathbb{R})^{\Omega}$, where $\phi(m)(\omega)$ is the DM's payment in state ω after she reports m. An *experiment* is a pair (D, ϕ). With a slight abuse of notation, I refer to an experiment simply as ϕ when D is clear from the context.

3.4 Preferences over Acts

Because each report yields an act rather than a single outcome, evaluating reports requires the DM's preference over acts. Let $\succsim^{\star}$ denote this preference over $(X \times \mathbb{R})^{\Omega}$, with asymmetric part $\succ^{\star}$, and call it an *extension* of $\succsim^{+}$. I impose the following assumptions on $\succsim^{\star}$.

Assumption 8 (Extension of Transitivity)**.** $\forall f, g, h \in X \times R^{\Omega}$, *$f \succsim^{\star} g$, and $g \succsim^{\star} h \Rightarrow f \succsim^{\star} h$.*

Assumption 9 (Extension of Reflexivity)**.** $\forall f \in X \times R^{\Omega}$, $f \succsim^{\star} f$

Assumption 10 (Agreement on Constant Acts)**.** *$\succsim^{\star}$ agrees with $\succsim$ on constant acts: for $(x, \eta), (y, \gamma) \in X \times \mathbb{R}$, $(x, \eta) \succsim^{+} (y, \gamma) \Leftrightarrow (x, \eta) \succsim^{\star} (y, \gamma)$.*

Assumption 11 (State Monotonicity)**.** *(i) $f(\omega) \succsim^{+} g(\omega)$ for all $\omega \in \Omega \Rightarrow f \succsim^{\star} g$, and if also $f(\omega) \succ^{+} g(\omega)$ for some ω, then $f \succ^{\star} g$. (ii) $f \succ^{\star} g \Rightarrow \exists \omega \in \Omega$, $f(\omega) \succ^{+} g(\omega)$.*

Part (i) of Assumption 11 is the state monotonicity condition of Azrieli et al. (2018). When $\succsim^{+}$ is complete, part (ii) follows from part (i): if $f(\omega) \not\succ^{+} g(\omega)$ for every ω, then $g(\omega) \succsim^{+} f(\omega)$ for every ω, so $g \succsim^{\star} f$ by part (i), and hence $f \not\succ^{\star} g$. Under completeness, Assumption 11 therefore reduces to their condition.

For each $\succsim^{+} \in \mathcal{R}$, let $\mathcal{C}(\succsim^{+})$ denote the set of extensions $\succsim^{\star}$ satisfying Assumptions 8–11.

4 Incentive Compatibility

4.1 Benchmark in Complete Preference

Experiments in the literature usually assume completeness of preferences. Therefore, I will start with the benchmark of incentive compatibility in experiments with complete preferences. Given the agreement assumptions (Assumption 5 and 10), it suffices to assume completeness in acts $X \times R^{\Omega}$ here.

Assumption 12 (Completeness)**.** $\forall f, g \in X \times R^{\Omega}$*, at least one of the following holds:* $f \succsim^{\star} g$ *or* $g \succsim^{\star} f$*.*

Under completeness, no two options are incomparable, so the incomparability message is not needed, and the message set in each D_i reduces to $M_i = \{d_{1i} \hat{\succsim} d_{2i},\ d_{2i} \hat{\succsim} d_{1i}\}$; the last case in the definition of $T_i(\succsim^{+})$ never arises. Moreover, the monetary supplements serve to separate indifference from incomparability, so they play no role here, and I take D to consist of core tasks only.

Definition 1 (Strict Incentive Compatibility with Complete Preferences (SICCP))**.** *A mechanism* ϕ *is* strictly incentive compatible *if, for every complete* $\succsim^{+} \in \mathcal{R}$*, every* $\succsim^{\star}$ *satisfying Assumptions 8–12, and every truthful* $m^{\star} \in T(\succsim^{+})$*:*

1. $\phi(m^{\star}) \succsim^{\star} \phi(m)$ *for every* $m \in M$*; and*
2. $\phi(m^{\star}) \succ^{\star} \phi(m')$ *for every untruthful* $m' \in M \setminus T(\succsim^{+})$*.*

SICCP is the classical notion of strict incentive compatibility: truth-telling yields a best act, and every lie yields a strictly worse one. The RPS mechanism, which draws one decision problem at random and pays the option the DM chose in it, satisfies *SICCP* (Azrieli et al., 2018).

4.2 Incentive Compatibility With Incomplete Preferences

Strict incentive compatibility in the sense of Definition 1 does not extend to incomplete preferences. This section identifies what can be attained instead. Starting from a weak notion, which requires only that no report yield a strictly better act than the truth, I add strict incentives one kind of lie at a time, and the answer differs across the two sides of truth-telling. Lies about comparable pairs can be strictly penalized, but only for DMs whose incomparability survives every monetary change smaller than a fixed resolution $b > 0$; without this restriction, no mechanism penalizes them. Rankings reported on incomparable pairs cannot be strictly penalized by any mechanism, even under a weaker notion of truth-telling. The resulting notion, b-robust comparable-side incentive compatibility, is therefore the frontier of incentive compatibility under incomplete preferences.

4.2.1 Weak Incentive Compatibility with Incomplete Preferences

Definition 2 (Weak Incentive Compatibility with Incomplete Preferences (WICIP))**.** *A mechanism* ϕ *is* weakly incentive compatible *if, for every* $\succsim^{+} \in \mathcal{R}$*, every* $\succsim^{\star} \in \mathcal{C}(\succsim^{+})$*, every truthful* $m^{\star} \in T(\succsim^{+})$*, and every* $m \in M$*,* $\phi(m) \not\succ^{\star} \phi(m^{\star})$*.*

WICIP is the counterpart of condition 1 of Definition 1. For a complete $\succsim^\star$, $\phi(m) \not\succ^\star \phi(m^\star)$ is equivalent to $\phi(m^\star) \succsim^\star \phi(m)$, so the two requirements coincide. Under incomplete preferences, WICIP is weaker: truth-telling need not yield a best act, but no report yields a strictly better one. WICIP places no penalty on lies; a mechanism that pays the same act whatever the report satisfies it. The strict incentives studied below are added on top of it.

A message profile m is *statewise maximal* under $\succsim^+$ if no message profile pays strictly more in any state. The set of such profiles is

$$\bar{M}(\phi, \succsim^+) = \big\{m \in M : \ \phi(m')(\omega) \not\succ^+ \phi(m)(\omega) \text{ for all } m' \in M \text{ and } \omega \in \Omega\big\}.$$

Lemma 1. *Let $\succsim^+ \in \mathcal{R}$ and $m \in M$. Then $\phi(m') \not\succ^\star \phi(m)$ for every $m' \in M$ and every $\succsim^\star \in \mathcal{C}(\succsim^+)$ if and only if $m \in \bar{M}(\phi, \succsim^+)$. Consequently, ϕ satisfies WICIP if and only if $T(\succsim^+) \subseteq \bar{M}(\phi, \succsim^+)$ for every $\succsim^+ \in \mathcal{R}$.*

Lemma 1 turns WICIP, a condition on acts, into a condition on outcomes state-wise: in every state, a truthful report must pay an outcome that is maximal under $\succsim^+$ among the outcomes that some report would pay in that state. Sufficiency follows from Assumption 11(ii), since an act can be strictly preferred only if it pays strictly more in some state. Necessity holds because, whenever another report pays strictly more in some state, some extension in $\mathcal{C}(\succsim^+)$ ranks its act strictly above the truthful act (Lemma A.1 in the Appendix).

4.2.2 Comparable-Side Incentive Compatibility

The first strict incentive concerns pairs the DM can compare. Recall that a comparable-side lie misreports such a pair on a core task: it reverses a strict preference or reports incomplete preference. Comparable-side incentive compatibility requires such a lie to be penalized.

Definition 3 (Comparable-side Incentive Compatibility (CSIC))**.** *A mechanism ϕ satisfies* comparable-side incentive compatibility *if, for every $\succsim^+ \in \mathcal{R}$ and every $m^\star \in T(\succsim^+)$:*

1. *$\phi(m) \not\succ^\star \phi(m^\star)$ $\forall \succsim^\star \in \mathcal{C}(\succsim^+)$, and $\forall m \in M$,.*
2. *if m' contains a comparable-side lie, $\forall \succsim^\star \in \mathcal{C}(\succsim^+)$ that can compare $\phi(m^\star)$ and $\phi(m')$, $\phi(m^\star) \succ^\star \phi(m')$; and $\exists \succsim^\star \in \mathcal{C}(\succsim^+)$ that can compare them.*

The condition (1) in CSIC (Definition 3) is the same as WICIP (Definition 2). Condition 2 then requires that every extension in $\mathcal{C}(\succsim^+)$ that can compare $\phi(m^\star)$ and $\phi(m')$ strictly prefer $\phi(m^\star)$, and that at least one extension compare them. This condition differs from condition 2 of SICCP in two respects. First, CSIC imposes strict penalties only on comparable-side lies;

under the completeness assumed in SICCP, every lie is a comparable-side lie, so the distinction does not arise there. Second, SICCP requires the truthful act to be strictly preferred to the act from any lie under every extension, whereas CSIC requires strict preference only from extensions that compare the two acts: no extension may rank the lie weakly above the truth, and at least one must rank the truth strictly above it.

The second difference is where incompleteness matters. A literal translation of SICCP would require $\phi(m^\star) \succ^\star \phi(m')$ under every extension in $\mathcal{C}(\succsim^+)$. Consider the extension $\succsim_0^\star$ defined by $f \succsim_0^\star g$ if and only if $f(\omega) \succsim^+ g(\omega)$ for all $\omega \in \Omega$. It satisfies Assumptions 8–11 and therefore belongs to $\mathcal{C}(\succsim^+)$. Under $\succsim_0^\star$, the literal translation requires the truthful report to pay a weakly better outcome than the lie in every state and a strictly better one in some state. If the two reports pay incomparable outcomes in some state, this requirement fails, and the failure stems from the incompleteness of the DM's preferences rather than from the design of the mechanism. Condition 2 of CSIC therefore requires strict preference only from extensions that compare the two acts, and it requires at least one such extension to exist, so that the penalty is not vacuous.

The definition does not require every admissible extension to strictly prefer $\phi(m^\star)$ to $\phi(m')$ since completeness is not assumed for extension $\succsim^\star$.

Lemma 2 (Statewise form of CSIC)**.** *ϕ satisfies CSIC if and only if, $\forall \succsim^+ \in \mathcal{R}$:*

(a) $T(\succsim^+) \subseteq \bar{M}(\phi, \succsim^+)$; and

(b) $\forall m'$ contains a comparable-side lie and $\forall m^\star \in T(\succsim^+)$, $\exists \omega \in \Omega$ with $\phi(m^\star)(\omega) \succ^+ \phi(m')(\omega)$.

Lemma 2 turns CSIC, a condition on acts, into a condition on outcomes, state by state. Condition (a) restates the characterization of WICIP in Lemma 1: in every state, a truthful report must pay an outcome that is maximal under $\succsim^+$ among the outcomes that some report would pay in that state. Condition (b) is new: against every report that makes a comparable-side lie, a truthful report must pay a strictly better outcome in at least one state, which may depend on the lie.

CSIC, however, is not attainable without a bound on the thickness incomparability. To see why, I first derive a necessary condition for CSIC and then show that it conflicts with incomparability that is arbitrarily thin.

Define coexistence for a pair of options $(x, 0)$ and $(y, 0)$. $(x, 0)$ and $(y, 0)$ coexists in a message m of an experiment ϕ if there exists two states ω_x and ω_y such that $\phi(m)(\omega_x) = (x, \eta^-)$ with $\phi(m')(\omega_x) = (y, \gamma^+)$ for some $m' \in M$, and $\phi(m)(\omega_y) = (y, \gamma^-)$ with $\phi(m'')(\omega_y) = (x, \eta^+)$

for some $m'' \in M$, for some $\eta^- \leq 0 \leq \eta^+$ and $\gamma^- \leq 0 \leq \gamma^+$, with at least one of the four inequalities strict. I call (ω_x, ω_y) a *coexistence pair*, and $|\eta^-|$, $|\eta^+|$, $|\gamma^-|$, and $|\gamma^+|$ its margins in the directions $-x$, $+x$, $-y$, and $+y$.

In words, in ω_x the mechanism pays x while y with weakly more money remains within reach, and in ω_y it pays y while x with weakly more money remains within reach. Therefore, x does not rule out y nor y rule out x.

The mechanism observes only the report, not the preference behind it, so a requirement on truthful reports binds at every report that some DM could make truthfully. Let $M_R \subseteq M$ denote the set of message profiles consistent with some true preference $\succsim^+ \in \mathcal{R}$: $M_R = \{m \in M : \exists \succsim^+ \in \mathcal{R} \text{ s.t. } m \in T(\succsim^+)\}$.

Proposition 1. *If ϕ satisfies CSIC, then for every $m \in M_R$, every core task $D_i = \{(x,0),(y,0)\} \in D_{core}$, if $(x,0)\hat{\bowtie}(y,0)$ is reported in m, then $(x,0)$ and $(y,0)$ coexist under message m in experiment (ϕ).*

Proposition 1 says that is a pair of $(x,0)$ and $(y,0)$ are incomparable, they must coexists by truth-telling. The two parts of the definition of coexistence deter different comparable-side lies. Coexistence itself deters a DM with a strict preference who reports incomplete preference. If $(x,0) \succ^+ (y,0)$ and she reports $(x,0)\hat{\bowtie}(y,0)$, she is paid (y,γ^-) in ω_y although (x,η^+), which she strictly prefers, was within reach; if $(y,0) \succ^+ (x,0)$, she is paid (x,η^-) in ω_x although (y,γ^+) was within reach. Each state deters one of the two strict preferences, which is why both are needed. The strict inequality deters a different DM: one who is indifferent between $(x,0)$ and $(y,0)$ and reports incomplete preference. Being paid y while x is within reach at the same monetary level costs her nothing. To penalize her, the mechanism needs a state in which the outcome she is paid falls short of an outcome within reach by a positive amount of money, and the strict inequality provides such a state.

The margins cannot be too large: intuitively, they cannot exceed the thickness of the DM's incomparability. For a DM with $(x,0) \bowtie^+ (y,0)$, define the *thickness of incomparability* in the directions $+x$ and $-x$ as

$$d^{\succsim^+}_{+x}(x,y) = \sup\{\nu \geq 0 : (x,\nu) \bowtie^+ (y,0)\}, \qquad d^{\succsim^+}_{-x}(x,y) = \sup\{\nu \geq 0 : (x,-\nu) \bowtie^+ (y,0)\},$$

and define $d^{\succsim^+}_{+y}(x,y)$ and $d^{\succsim^+}_{-y}(x,y)$ symmetrically by varying the money attached to y. Each measures how far the money attached to one option can move in one direction before the two options become comparable. By Assumptions 3, 6, and 7, each of these sets is an interval that contains 0 and is open in $[0,\infty)$, so every thickness is strictly positive, possibly infinite.

Corollary 1. *If ϕ satisfies CSIC, for a DM with $(x,0) \bowtie^+ (y,0)$ and report $m^\star \in T(\succsim^+)$, the coexistence margin is smaller than the thickness of incomparability on each direction: $|\eta^-| \leq d^{\succ^+}_{\sim -x}(x,y)$, $|\eta^+| \leq d^{\succ^+}_{\sim +x}(x,y)$, $|\gamma^-| \leq d^{\succ^+}_{\sim -y}(x,y)$, and $|\gamma^+| \leq d^{\succ^+}_{\sim +y}(x,y)$.*

If one of these inequalities fails, the coexistence pair moves money far enough to make the two options comparable. Suppose, for instance, that $|\eta^-| \geq d^{\succ^+}_{\sim -x}(x,y)$. Then (x,η^-) and $(y,0)$ are comparable, and since $(x,0) \bowtie^+ (y,0)$, Monotonicity in Money and transitivity give $(y,0) \succ^+ (x,\eta^-)$. Because $(y,\gamma^+) \succsim^+ (y,0)$, the outcome (y,γ^+), which is within reach in ω_x, is strictly better than the outcome (x,η^-) that the truthful report pays there. This violates condition (a) of Lemma 2. The other three cases are symmetric.

Proposition 1 and Corollary 1 pull the margins of a coexistence pair in opposite directions. The strict inequality in the definition of coexistence keeps at least one of the four margins of every coexistence pair positive, because otherwise an indifferent DM could report incomparability at no cost. In a finite experiment, the smallest positive coexistence margin cannot be arbitarily small. While Corollary 1 caps each margin at the thickness of incomparability, which is unknown to experimenter and therefore can be arbitrarily small.

Proposition 2. *If $D_{core} \neq \emptyset$, no mechanism satisfies CSIC.*

Proposition 2 forces a choice. A mechanism must either stop penalizing indifferent DMs who report incomparability, or stop protecting DMs whose incomparability is very thin. We take the second route and fix a resolution $b > 0$. Let

$\mathcal{P}_b = \Big\{ \succsim^+ \in \mathcal{R} : \; d^{\succsim^+}\big((x,0),(y,0)\big) \geq b \;$ for every $\{(x,0),(y,0)\} \in D_{core}$ with $(x,0) \bowtie^+ (y,0) \Big\}$, where $d^{\succsim^+}\big((x,0),(y,0)\big)$ is the smallest thickness of incomparability among the four directions.

Definition 4 (b-Robust Comparable-Side Incentive Compatibility (b-RCSIC))**.** *A mechanism ϕ satisfies* b-robust comparable-side incentive compatibility *if conditions 1 and 2 of Definition 3 hold for every $\succsim^+ \in \mathcal{P}_b$ and every truthful $m^\star \in T(\succsim^+)$.*

The parameter b is the resolution at which the experiment separates incomparability from indifference. b-RCSIC restricts attention to DMs whose incomparability is thick enough for the mechanism to let the two elements coexist without violating their true preference.

b-RCSIC can be attained. A specific mechanism using Random Incentive System that attain it is shown in the appendix.

4.2.3 Incomparable-side Incentive Compatibility

Recall that an incomparable-side lie misreport a incomparable pair as comparable. The Incomparable-side Incentive Compatibility requires incomparable-side lies to be penalized.

Definition 5 (Incomparable-side Incentive Compatibility (ISIC))**.** *A mechanism ϕ satisfies* incomparable-side incentive compatibility *if, for every $\succsim^+ \in \mathcal{R}$ and every $m^\star \in T(\succsim^+)$:*

1. *$\phi(m) \not\succ^\star \phi(m^\star)$ $\forall \succsim^\star \in \mathcal{C}(\succsim^+)$, and $\forall m \in M$,.*
2. *if m' contains an incomparable-side lie, $\forall \succsim^\star \in \mathcal{C}(\succsim^+)$ that can compare $\phi(m^\star)$ and $\phi(m')$, $\phi(m^\star) \succ^\star \phi(m')$; and $\exists \succsim^\star \in \mathcal{C}(\succsim^+)$ that can compare them.*

ISIC (Definition 5) is the counterpart of CSIC (Definition 3) that penalizes incomparable-side lie.

Lemma 3. *A mechanism ϕ satisfies ISIC if and only if, for every $\succsim^+ \in \mathcal{R}$:*

(a) $T(\succsim^+) \subseteq \bar{M}(\phi, \succsim^+)$;

(b) for every core task $\{(x,0),(y,0)\}$ with $(x,0) \bowtie^+ (y,0)$, every report m' that declares $(x,0) \hat{\succsim} (y,0)$ or $(y,0) \hat{\succsim} (x,0)$ on it, and every $m^\star \in T(\succsim^+)$, there is $\omega \in \Omega$ with $\phi(m^\star)(\omega) \succ^+ \phi(m')(\omega)$.

Proposition 3. *If $D_{core} \neq \emptyset$, no mechanism satisfies ISIC.*

The intuition behind the impossibility results is that a DM with incomplete $\succsim_0^+$ can report a completion of her preferences $\succsim_1^+$ such that $\succsim_1^+$ is complete and $\succsim_1^+$ preserves all the strict and weak preferences in $\succsim_0^+$. Such completion exists using theory of order. Therefore, whatever mechanism that penalize an incomparable-side lie for this DM will also penalize another DM whose true preferences is $\succsim_1^+$.

Since strict incentives for incomparability reporting are unattainable, we consider a even weaker notion of truth-telling. Partition M into:

$$M_c = \{m \in M : m_i \in \{d_{1i} \hat{\succsim} d_{2i},\, d_{2i} \hat{\succsim} d_{1i}\},\, \forall D_i \in D_{core}\},$$

$$M_{ic} = M \setminus M_c,$$

where M_c contains all message profiles that report a complete ordering on every core task, and M_{ic} contains all profiles that report incomparability in at least one core task.

We say that $\succsim^+$ is *complete on D_{core}* if every core task is comparable under $\succsim^+$, that is, there is no $D_i \in D_{core}$ with $d_{1i} \bowtie^+ d_{2i}$; otherwise $\succsim^+$ is *incomplete on D_{core}*. Rather

than requiring truthful reporting of the exact preference relation, *weak truth-telling* only requires that the DM reports in the correct partition: a DM with complete $\succsim^+$ on D_{core} should report some $m \in M_c$, and a DM with incomplete $\succsim^+$ on D_{core} should report some $m \in M_{ic}$. The following definition asks whether a mechanism can strictly incentivize even this weaker requirement on the incomparable side, that is, whether it can ensure that a DM with incomplete preferences strictly prefers some $m \in M_{ic}$ over any $m \in M_c$, while only weakly incentivizing the analogous requirement on the comparable side.

Definition 6 (Incomparable-side Incentive Compatibility for Weak Truth-telling (ISICWT))**.** *A mechanism ϕ satisfies* incomparable-side incentive compatibility for weak truth-telling *if, for every $\succsim^+ \in \mathcal{R}$:*

1. *If $\succsim^+$ is complete on D_{core}, then for every $\succsim^\star \in \mathcal{C}(\succsim^+)$ there is $m \in M_c$ with $\phi(m') \not\succ^\star \phi(m)$ for all $m' \in M_{ic}$.*
2. *If $\succsim^+$ is incomplete on D_{core}, then for every $\succsim^\star \in \mathcal{C}(\succsim^+)$ there is $m \in M_{ic}$ with $\phi(m') \not\succ^\star \phi(m)$ for all $m' \in M_c$.*
3. *If $\succsim^+$ is incomplete on D_{core}, there is $m \in M_{ic}$ such that for every $m' \in M_c$ some $\succsim^\star \in \mathcal{C}(\succsim^+)$ has $\phi(m) \succ^\star \phi(m')$.*

Conditions 1 and 2 are weak requirements; condition 3 asks for a strict incentive on the incomparable side. Each condition is implied by ISIC: a truthful report lies in the correct partition, and when $\succsim^+$ is incomplete on D_{core} every $m' \in M_c$ is an incomparable-side lie. ISICWT is therefore weaker than ISIC, and Proposition 4 implies Proposition 3.

Proposition 4. *If $D_{core} \neq \emptyset$, no mechanism satisfies ISICWT.*

The intuition of the impossibility result of ISICWT is the same as ISIC.

Proposition 4 shows that strict incentives for incomparability reporting are unattainable even under the weaker notion of truth-telling. Combined with Proposition 3, this establishes that no mechanism can strictly incentivize truth-telling on the incomparable side.

4.2.4 Frontier of Incentive Compatibility with Incomplete Preferences

The results above establish the following picture. Weak incentive compatibility (Definition 2) is the foundation. On top of it, comparable-side lies can be penalized, subject to a lower bound on the thickness of incomparability in D_{core} (b-RCSIC, Definition 4), while the incomparable-side lies cannot be penalized (Propositions 3 and 4). Therefore, b-RCSIC is the frontier of incentive compatibility under Assumptions 1–11. This frontier implies that

the proportion of reported incomplete preferences in an experiment is a lower bound of the real proportion and we can only identify incomplete preferences in a conservative ways.

5 Attaining the Frontier with a Sequential Mechanism.

The mechanisms of Section 4 use a random incentive system, and their incentive properties rest on State Monotonicity of the extension $\succsim^\star$ (Assumption 11), following Azrieli et al. (2018). Because State Monotonicity is a property of $\succsim^\star$, one might expect it to place no restriction on the DM's preference $\succsim^+$ over outcomes. This expectation fails in the domains of risk and ambiguity. When the objects in X are lotteries and the DM reduces each act, a compound lottery, to a simple lottery, State Monotonicity requires $\succsim^+$ to satisfy independence (see also Azrieli et al., 2018; Freeman et al., 2019), so a mechanism justified by State Monotonicity is incentive compatible only for DMs whose preferences satisfy independence. When the objects are ambiguous prospects, an ambiguity-averse DM may instead value the random device as a hedge, which violates State Monotonicity (Baillon et al., 2022).

This section reaches the same frontier, b-RCSIC, with a sequential mechanism that uses no random device and the report of DMs pin down the final outcome. In addition, options are revealed one at a time, and the DM cannot foresee the options still to come. These two features mitigates the reduction and hedging effects described above. I start from the Accumulative Best Choice (ABC) mechanism of Y. Li (2021) that is incentive compatible for every complete and transitive preference. I extend the message space to allow the report of incomplete preferences and also extend single list to multiple lists to attain b-RCSIC.

5.1 The ABC Mechanism and Its Extension

The ABC mechanism. Fix a list $L = (l_0, l_1, \ldots, l_N)$ of elements of $X \times \mathbb{R}$, revealed one element per round. The DMs starts with holding the first option. In each round $i = 1, \ldots, N$ the DMs are asked to compare the option they are holding, written as c_{i-1}, with the option l_i and report one of the two rankings: $c_{i-1} \hat{\succsim} l_i$ or $l_i \hat{\succsim} c_{i-1}$, and the reported preferred option becomes c_i and is carried forward to the next round to be compared with the next option l_{i+1}. The process continues until the last round. The payoff is c_N, the last option they hold.[4] ABC mechanisms has three good features: (i) The list is fixed in advance and revealed sequentially, so the DM can neither foresee nor influence future elements. (ii) Each decision task $D_i = \{c_{i-1}, l_i\}$ is determined by the DM's own earlier reports rather than by an exogenous random device. Hence, every including c_N, is determined by the report alone,

[4]This paper focuses on binary menu while the menu can be any size in ABC mechanisms.

with no random device. (iii) Under truthful reporting c_i is a best option among $\{l_0, \ldots, l_i\}$ in every round, so c_N is a best choice of the list.

ABC mechanism is for complete preferences only. I extend the *ABC* mechanism for incomplete preferences. The DM may report $c_{i-1} \hat{\bowtie} l_i$ alongside with $c_{i-1} \hat{\succsim} l_i$ and $l_i \hat{\succsim} c_{i-1}$. The mechanism decides which option is carried forward to the next round after the report of $c_{i-1} \hat{\bowtie} l_i$. The mechanism can retain c_{i-1}, switch to l_i, switch back to some earlier option l_j, $j < i$, or ask further comparisons before settling c_i. I allow all different designs, except for the use of random devices, to decide c_i. In addition, the length of the list L can be unknown to the DMs. In the ABC mechanism, whether N is known does not impact the incentive compatibility. Here we allow the length of L to be unknown.

An *extended ABC mechanism* ϕ runs as follows. Round i opens with the comparison $D_i^1 = \{c_{i-1}, l_i\}$. If the DM declares a complete preferences, the round ends and c_i is the declared-preferred element, as in *ABC*. If she declares $\hat{\bowtie}$, the mechanism may present further comparisons $D_i^2, D_i^3, \ldots$, each a pair of elements of $\{l_0, \ldots, l_i\}$, and then sets c_i to an element of $\{l_0, \ldots, l_i\}$. Which comparison comes next, when to stop, and which element becomes c_i are fixed in advance as deterministic functions of all declarations made so far. The payment is c_N. The *ABC* mechanism is the special case in which $\hat{\bowtie}$ is not available.

5.2 Incentive Compatibility for Extended ABC Mechanisms

Given the different experiment structure, the counterpart of b-RCSIC (Definition 4) for extended *ABC* mechanisms needs to be defined.

In an extended *ABC* mechanism, a report sequence m records the DM's declarations in the order in which the tasks are presented, and the payment $\phi(m) = c_N$ is determined by these declarations alone. Incentive compatibility can therefore be stated directly in terms of the primitive $\succsim^+$, without an extension $\succsim^\star$ to acts. Because the options still to come are unrestricted, I assume that the DM cannot rule out any continuation of the list. In particular, I exclude a DM who believes that certain options will appear later, and a DM who holds a probability distribution over the composition of the list. The latter may evaluate her payment as a mapping from list compositions to outcomes, that is, as an act, and the concerns raised at the beginning of this section would return. I discuss this limitation at the end of the section.

Throughout this section, whenever the outcomes of two reports, $\phi(m)$ and $\phi(m')$, are compared, it is assumed that both report are answered in the same L. A *core task* is a presented pair $\{(x, 0), (y, 0)\}$ in which both options carry zero money, D_{core} is the set of such pairs

that can be presented under some list and some reports, and $\mathcal{P}_b$ is defined as in Section 4 with this D_{core}.

Definition 7 (b-RCSIC for Extended ABC Mechanisms)**.** *An extended ABC mechanism ϕ is* b-robust comparable-side incentive compatible *(b-RCSIC) if, for every $\succsim^+ \in \mathcal{P}_b$:*

1. *$\phi(m) \not\succ^+ \phi(m^\star)$ $\forall m$ and $\forall m^\star$;*
2. *$\forall m$ that contains a comparable-side lie in any round i, there is a continuation of list after round i such that, $\phi(m^\star) \succ^+ \phi(m')$ $\forall m^\star \in T(\succsim^+)$ and $\forall m'$ that agrees with m up to and including the lie.*

Conditions 1 and 2 are the counterparts of the two conditions of b-RCSIC in Definition 4. Condition 1 requires that truth-telling is not worse than any other report. Condition 2 requires that every comparable-side lie be penalized: after a lie in round i, there is at least one continuation of the list on which the DM ends with a strictly worse payment than under truth-telling, no matter how she reports in the remaining tasks. That is, any m with a comparable-side lie is weakly dominated by truth-telling. The condition does not require the penalty on every continuation because no extended *ABC* mechanism could do it. For instance, if the final option is strictly better than every option revealed so far, a DM who lies and then reports truthfully in the final round ends with that option, exactly as a truthful DM does.

5.3 Impossibility with a Single List

Proposition 5. *For any $b > 0$, no extended ABC mechanism with a single list L is b-RCSIC.*

Proposition 5 shows that with a single list, no extended ABC mechanism attains b-RCSIC, however flexibly it responds to reports of incomparability. To explain the intuition behind this impossibility result, we first introduce a necessary condition for an extended ABC mechanism to be b-RCSIC and then use it to illustrate the contradiction.

For a set S of outcomes, let $\text{Maximal}(S, \succsim^+) = \{s \in S : \ t \succ^+ s \text{ for no } t \in S\}$. An extended ABC mechanism is an *Accumulative Maximal Choice* (AMC) mechanism if truthful reporting keeps the current choice maximal among the options revealed so far: for every L, every $\succsim^+ \in \mathcal{P}_b$, every $m \in T(\succsim^+)$, and every round i, $c_i(m) \in \text{Maximal}\big(\{l_0, \ldots, l_i\}, \succsim^+\big)$.

Proposition 6. *Every b-RCSIC extended ABC mechanism is an AMC mechanism.*

The intuition for the necessary condition is straightforward. If c_i is not maximal in $\{l_0, \ldots, l_i\}$, some element already seen is strictly better than c_i under the DM's true preference $\succsim^+$. If

this is the final round or every future option is strictly worse than c_i, the truthful payment is c_i, strictly worse than what the mechanism could have delivered, violating condition 1 of Definition 7.

Suppose that c_{i-1} is maximal in $\{l_0, \ldots, l_{i-1}\}$ and incomparable with l_i. Then c_{i-1} is maximal in $\{l_0, \ldots, l_i\}$, but the same need not hold for l_i or for any earlier option l_j with $j < i$ and $l_j \neq c_{i-1}$: l_i may be strictly worse than an earlier option l_j, or l_j may be strictly worse than l_i. There are two natural ways to build an AMC mechanism: (i) keep c_{i-1} whenever $c_{i-1} \hat{\bowtie} l_i$ is reported, or (ii) compare l_i with the relevant earlier options, setting c_i to l_i if l_i is not declared worse than any of them and keeping c_{i-1} as c_i otherwise. Both fail b-RCSIC. In (i), the reports $c_{i-1} \hat{\succsim} l_i$ and $c_{i-1} \hat{\bowtie} l_i$ lead to the same path, so a DM with $c_{i-1} \succsim^+ l_i$ can report $c_{i-1} \hat{\bowtie} l_i$ without being penalized. In (ii), suppose that some earlier option l_j is incomparable with c_{i-1} and that both are strictly preferred to l_i. A DM with these preferences can report $c_{i-1} \hat{\bowtie} l_i$, knowing that this report triggers a comparison between l_j and l_i, and then report $l_j \hat{\succsim} l_i$ to rule out l_i. The first report is a comparable-side lie, yet the non-maximal option l_i is still ruled out and c_{i-1} is carried forward.

In sum, in single list, a DM who strictly prefers c_{i-1} over l_i can report incomplete preference and still rule out l_i. Since no single-list mechanism is b-RCSIC, the next subsection extends the mechanism from a single list to multiple lists.

5.4 Multiple Lists

In single list, a *AMC* mechanism is necessary but not enough. Therefore, I extend *AMC* mechanisms to *Multi-list AMC* mechanism. Let me first introduce what do I mean by multiple list.

Instead of a single list, now each experiment contains multiple lists instead of just one list. A *multi-list AMC mechanism* runs an AMC mechanism on each list, under three rules.

1. *Reset.* Each list starts afresh and the first current option is set as the first option in this new list $c_0 = l_0$. Without this reset rule, multiple lists is equivalent to a single list.

2. *Binding rule.* (i) If a pair of comparisons appears again, the earlier reports binds and cannot be changed [5]. (ii) A report of incomplete preference, $(x, 0) \hat{\bowtie} (y, 0)$, is recorded together with $(x, \eta') \hat{\bowtie} (y, 0)$ and $(x, 0) \hat{\bowtie} (y, \gamma')$ for all $|\eta'| < b$ and $|\gamma'| < b$, where the threshold b is chosen by the experimenter.

[5] A variation is to allow the DM to revise a prior declaration at a penalty, which preserves the binding structure while accommodating preference refinement.

3. *Unknown structure and options.* The number of lists K, their lengths N_k, and their contents are not known to the DM in advance.

Write $L^1, \ldots, L^K$ for the lists, in the order presented. The payment is the final current choice of the last list L^K, $c^K_{N_K}$. A continuation play following every round i consists of the future options in the current list together with the lists that are still to come.

Multiple lists can achieve what single list cannot. As discussed above, in a single list, a DM who strictly prefers c_{i-1} over l_i can report incomplete preference and still rule out l_i. However, in the multiple lists structure, there could be more lists coming and the order and content of future lists is unknown. If the same pair appears again, option l_i may not be ruled out again given the binding rule.

Proposition 7. *b-RCSIC can be attained by some* Multi-list AMC *mechanisms.*

Proposition 7 shows that *Multi-list AMC* mechanisms can attain *b*-RCSIC. In particular, two *Multi-list AMC* mechanisms that built on the two ways to *AMC* mechanisms discussed in the Section 5.3 are both *b*-RCSIC both work. Let me formally define the two *AMC* mechanisms.

AMC-1 (status quo rule)

- *AMC-1 (status quo rule).* If $c_{i-1} \hat{\succsim} l_i$ or $c_{i-1} \hat{\bowtie} l_i$ is reported, then $c_i = c_{i-1}$; if $l_i \hat{\succsim} c_{i-1}$ is reported, then $c_i = l_i$.

AMC-2 (Buffer Set Rule)

- The mechanism keeps a buffer $W_i \subseteq \{l_0, \ldots l_i\}$, with $W_0 = \emptyset$ in the beginning of each list. In round i:
  1. if $c_{i-1} \hat{\succsim} l_i$ is reported, then $c_i = c_{i-1}$ and $W_i = W_{i-1}$;
  2. if $l_i \hat{\succsim} c_{i-1}$ is reported, then $c_i = l_i$ and $W_i = W_{i-1}$;
  3. if $c_{i-1} \hat{\bowtie} l_i$ is reported, the mechanism presents $\{w, l_i\}$ for the elements $w \in W_{i-1}$, one at a time. If some $w \hat{\succsim} l_i$ is reported, the round ends with $c_i = c_{i-1}$ and $W_i = W_{i-1}$. If no such declaration occurs, $c_i = l_i$ and $W_i = W_{i-1} \cup \{c_{i-1}\}$.

Refer to the resulting mechanisms as *Multi-list AMC-1* and *Multi-list AMC-2*; Let's continue to use the example in the Section 5.3 to illustrate why they are *b*-RCSIC. A DM prefers c_{i-1} to l_i and if he reports $c_{i-1} \hat{\bowtie} l_i$, this will lead to a penalization in some scenarios no matter what she reports in the future lists. Since the DM knows neither the number nor the length of the lists, the current list may end right after a lie and be followed by a single final list. Let $c_{i-1} = (x, 0)$ and $l_i = (y, 0)$. Under *Multi-list AMC-1*, let the final list be $((y, 0), (x, \eta))$ where

$-b > \eta > 0$: by the reset rule $(y, 0)$ is the initial current choice, and the binding rule applies here, therefore $(y, 0)$ is kept and becomes the final payoff, whereas truth-telling report leads to (x, η), and $(x, \eta) \succ^{+} (y, 0)$. Under *Multi-list AMC-2*, let the final list be $((x, \eta), (y, 0)))$: the buffer is empty at the start of the list, so $(y, 0)$ is carried forward and becomes the final payoff, while truthful reports leads to (x, η). Section 6 evaluates both mechanisms in the laboratory.

5.4.1 Advantages and Limitations of *Multi-list AMC* Mechanisms

As discussed before, the motivation to use sequential design instead of a Random Incentive system is to solve the problem from reduction of compound lottery in the domain of risk and hedging in the domain of ambiguity. The sequential mechanism can mitigate the problem given that DMs cannot foresee future options and no random device is used in the experiment. Therefore, *Multi-list AMC* mechanisms are applicable to any domain.

However, there is also limitation based on the belief of DMs on the future options. Firstly, if a DM believes that better option is definitely will appear in the final list than the options in the current round, the answer in the current round does not matter and this belief will undermines the incentive for truth-telling. Secondly, even though the future options are unknown, if a DM forms a subjective belief and has a probability distribution over the composition of future options and view the payoff as a mapping from the composition to outcomes based on the probability distribution, the same issue with Random Incentive System will appear here. To reduce the chance such circumstances happen, I do not tell the participants the average money they can earn in the experiment and they are told that the compositions of each list are predetermined, their report will not impact what options they will see in each list.

6 Experimental Design

6.1 Overview

In the experiment, each subject works through 7 lists in total. Each list contains three to five options. Each option is either a risky or ambiguous prospect plus a certain sure amount of money. I choose the domain of risk and ambiguity, as random payment systems have shown problematic in these two domains Freeman et al. (2019) & Baillon et al. (2022). Within each list, options are numbered in the order in which they first appear: the option the subject begins holding is Option 1, and every new option takes the next number. Numbering restarts

at one in each list, so a number carries no information from one list to the next. The 7 lists are identical for each subject while the order is randomized across subjects.

The experiment uses a 3*2=6 arms between subject design. The first factor is the *mechanisms*, which fixes what a subject is allowed to say about a pair and what happens to the option set once they say it. There are three: *Multi-list ABC*, *Multi-list AMC-1* and *AMC-2*. In this experiment, I choose b as 0.25 CAD [6].The binding rule with this threshold applies to every comparison, not only to core tasks.[7] The second factor is the presence of a verification round [8]. In the plain arms the experiment ends with the seventh list. In the verification arms a single further round follows, repeating one comparison the subject has already answered and offering a straight choice between its two options. Subjects are told that this is the final round and that the option they choose determines their payment. Therefore, the verification round is equivalent to a single-round experiment, and in a single-round experiment, truthful choice is incentive compatible if subjects can rank the two options (Azrieli et al. (2018)). Consistency with this benchmark is viewed as a standard test of a mechanism's incentive compatibility. The verification round serves as a within subject consistency test and lets me compare a subject's report inside the mechanism with their choice in the benchmark.

In the *Multi-list ABC*, each comparison of Option A and Option B offers two answers:

- I rank option A above option B
- I rank option B above option A

In the *Multi-list AMC-1* and *Multi-list AMC-2*, each comparison of Option A and Option B offers four answers:

- I rank option A above option B
- I rank option B above option A
- I rank the two options exactly the same

[6]Canadian coins have dimes. The smallest unit of payable money is 0.1 CAD. I choose 0.25 CAD to target more robust elicited incomplete preferences.

[7]Section 5.4 states the binding rule for core tasks only. I apply it to all comparisons for two reasons. First, a single rule for every comparison is easier for subjects to understand. Second, the broader rule requires only a small extension of the thickness condition in Definition 7. Because a sure payment is paid in every state, a lottery with sure payment s is the same prospect as the lottery that adds s to every prize and pays no sure amount. Relabeled in this way, X, Y and Z_2 in Table 3 carry no sure payment, and the only two options that differ solely in their sure payment, by less than b, are Z_1 and Z_2. Since Monotonicity in Money ranks the three Z options against one another, the broader rule affects a truthful subject only through a declaration that Z_1 cannot be ranked against X or Y, which then binds for Z_2 as well. It therefore suffices to extend the condition $d^{\succsim^+} \geq b$ to the pairs $\{X, Z_1\}$ and $\{Y, Z_1\}$.

[8]The verification round is inspired by Calford and Chakraborty (2025).

- I do not know how I rank Option A and Option B

The *Multi-list ABC* only include two answers to mimic the forced choice method widely used in the literature. If a subject is indifferent between the two options, any answer is a truthful report.

The two further answers available under *Multi-list AMC-1* and *Multi-list AMC-2* record indifference and incomparability respectively. "I rank the two options exactly the same" means indifference and "I do not know how I rank Option A and Option B" means incomplete preference. The theoretical framework above include only three: the two weak preferences and a report of incomplete preference, since indifference is simply the case in which both weak preferences hold. I give indifference its own answer in the experiment so that a subject who ranks the two options exactly equal is not confused and choose "I do not know how I rank Option A and Option B". One caveat follows. The mechanisms impose no cost on an indifferent subject who reports a strict preference by choosing one of the first two answers rather than a tie. A report of one of the first two answers under *Multi-list AMC-1* or *Multi-list AMC-2* is therefore consistent with either a strict preference or indifference.

The 7 lists are built so that the comparisons inside them test expected utility through the common-ratio pattern (List 1-2), subjective expected utility through the three-colour Ellsberg pattern (List 3-4), and transitivity (List 5-7) directly. Therefore, the relationship between incomplete preferences and the violations of EUT, SEU, and transitivity can be investigated. This design lets me examine how incomplete preferences relate to violations of EUT, SEU, and transitivity. This question is exploratory and secondary to the main purpose of the experiment, which is to evaluate whether the mechanisms are incentive compatible in behaviour.

Table 1 shows options in List 1 and List 2.

Setting the \$0.20 aside, the four bets form a common-ratio pair: a 25 percent chance of \$6 against a 20 percent chance of \$8 at the low stage, and the same two bets with every probability multiplied by four at the high stage. List 1 adds the \$0.20 to B_1 and A_2; List 2 adds it to A_1 and B_2 instead.

Both lists follow an adaptive route. Put aside C and D for now. A subject begins holding A_1, and the first comparison is A_1 against B_1. Whichever option they are left holding after the first round fixes the rest of the route. A subject holding A_1 next meets A_2, and a subject holding B_1 next meets B_2: in each case the new option is the high version of the bet they

Option	Urn	Prize if the ball is Red	Blue	Yellow	Sure payment
Panel A. List 1					
A_1	25 R, 75 B	6.00	0.00	0.00	0.00
B_1	20 R, 80 B	8.20	0.00	0.00	0.00
A_2	100 R	6.20	0.00	0.00	0.00
B_2	80 R, 20 B	8.00	0.00	0.00	0.00
C	20 R, 80 B	1.00	0.00	0.00	0.00
Panel B. List 2					
A_1	25 R, 75 B	6.20	0.00	0.00	0.00
B_1	20 R, 80 B	8.00	0.00	0.00	0.00
A_2	100 R	6.00	0.00	0.00	0.00
B_2	80 R, 20 B	8.20	0.00	0.00	0.00
D	20 R, 80 B	3.00	0.00	0.00	0.00

Table 1: Options in List 1 and List 2

are currently holding, which I call the *upgrade*. Almost every subject takes it [9]. A subject who does then meets the high version of the other bet, so a subject on the A arm finishes against B_2 and a subject on the B arm against A_2. Either way the route delivers the two core comparisons the list is built to measure, A_1 against B_1 and A_2 against B_2.

By adding the \$0.20 to different options in lists 1 and 2, I can identify a violation of expected utility from reports that are only weak rankings. Expected Utility Theory (**EUT**) is violated whenever the subject:

- in list 1, ranks A_1 above or exactly the same as B_1, and ranks B_2 above or exactly the same as A_2 (this is consistent with the Allais Paradox);
- in list 2, ranks B_1 above or exactly the same as A_1, and ranks A_2 above or exactly the same as B_2 (this is reverse to the Allais Paradox).

C and D are dominated options, inserted into the routes of lists 1 and 2 respectively. C pays \$1 and D pays \$3, each on a 20 percent chance, so each is first-oder stochastically dominated by every option in the lists. If subjects respect *FOSD*, they should rank the dominant option above the dominated option. Therefore, in these dominance comparisons, the true preference is already known and it can tests whether subjects report truthfully

[9] Of the 844 routes through Lists 1 and 2 (422 subjects, two lists each), 842 reach the upgrade; in the other two, the subject took the dominated option D in List 2 before reaching it. In 821 of these 842 comparisons (97.5%), the subject ranks the upgrade above the bet they hold. In 19 (2.3%) they rank the bet they hold above the upgrade, in one they report the two as exactly the same, and in one they report that they cannot rank them.

Option	Urn	Prize if the ball is Red	Blue	Yellow	Sure payment
Panel A. List 3					
A_1	30 R, 60 B/Y	6.00	0.00	0.00	0.00
B_1	30 R, 60 B/Y	0.00	6.20	0.00	0.00
A_2	30 R, 60 B/Y	6.20	0.00	6.20	0.00
B_2	30 R, 60 B/Y	0.00	6.00	6.00	0.00
E	30 R, 60 B/Y	0.00	0.00	0.00	0.00
Panel B. List 4					
A_1	30 R, 60 B/Y	6.20	0.00	0.00	0.00
B_1	30 R, 60 B/Y	0.00	6.00	0.00	0.00
A_2	30 R, 60 B/Y	6.00	0.00	6.00	0.00
B_2	30 R, 60 B/Y	0.00	6.20	6.20	0.00
F	30 R, 60 B/Y	0.00	0.00	2.00	0.00

Table 2: List 3-4

according to their preference. In addition, adding a dominated options avoid subjects forms a belief that future options are always better. Without a dominated option a subject could come to expect every arriving option to improve on the one they hold, and stop weighing the comparison in front of them.

Table 2 shows options in lists 3 and 4.

In lists 3 and 4, every option is a bet on a single urn of 90 balls, of which 30 are red and the remaining 60 are blue or yellow in a proportion the subject is not told. A bet on red therefore wins with known probability 1/3 and a bet on blue or yellow with known probability 2/3, while a bet on blue alone or on red or yellow is ambiguous. List 3 adds the \$0.20 to B_3 and A_4, the two ambiguous bets; list 4 adds it to A_3 and B_4, the two unambiguous ones.

By adding the \$0.20 to different options in lists 3 and 4, I can identify a violation of expected utility from reports that are only weak rankings. Subjective expected utility (**SEU**) is violated whenever the subject

- in list 3, ranks A_1 above or exactly the same as B_1, and ranks B_2 above or exactly the same as A_2 (ambiguity averse);
- in list 4, ranks B_1 above or exactly the same as A_1, and ranks A_2 above or exactly the same as B_2 (ambiguity seeking).

Lists 3 and 4 adopts the same adaptive route as Lists 1 and 2. Therefore, the route delivers the two core comparisons the list is built to measure, A_3 against B_3 and A_4 against B_4.

Option	Urn	Prize if the ball is			Sure payment
		Red	Blue	Yellow	
X	30 R, 30 B, 30 Y	10.00	10.00	2.00	0.20
Y	30 R, 30 B, 30 Y	18.00	3.00	3.00	0.20
Z_1	30 R, 30 B, 30 Y	6.00	6.00	6.00	0.00
Z_2	30 R, 30 B, 30 Y	6.00	6.00	6.00	0.20
Z_3	30 R, 30 B, 30 Y	6.00	6.00	6.00	0.50

Table 3: List 5-7

Option E is dominated by all the all options in list 3 and is inserted into list 3. Option F is first-order stochastically dominated by $A4$ and $B4$ and is the last option in list 4. The purpose of E and F is the same as C and D in lists 1 and 2.

Table 3 shows options in lists 5, 6, and 7.

In lists 5-7, all five options draw from one urn of 90 balls, 30 of each colour, so each colour is therefore drawn with probability 1/3. Z_1, Z_2 and Z_3 are riskless and differ only in the sure payment. X carries medium risk and a medium return, Y high risk and a high return. No one of X, Y and any Z dominates another. The three lists present these options in orders and do not use adaptive route. List 5 runs X, Y, Z_1, Z_3; list 6 runs Z_2, Z_1, X, Z_3; and list 7 runs Z_2, Y, Z_3. In each the subject begins holding the first option and meets the others in turn. Therefore, List 5 puts X against Y for every subject in the first round. In list 6, the subject is holding Z_2 or Z_1 when X arrives in the second round, and List 7 puts Z_2 against Y in the first round. So every subject ranks X against a Z, Y against a Z, and X against Y, which are the core comparisons in lists 5-7 and they are what makes a cycle detectable.

By using Z with different levels of sure amount of payment, I can identify a violation of expected utility from reports that are only weak rankings. With $i > j$, transitivity is violated by:

- ranks X above or exactly the same as Z_i, ranks a Z_j above or exactly the same as Y, and ranks Y above or exactly the same as X; or
- ranks Y above or exactly the same as Z_i, ranks Z_j above or exactly the same as X, and ranks X above or exactly the same as Y.

In addition, one further comparison is inserted following a random round from the twenty-four comparisons of the seven lists. It puts the currently holding option against a copy of itself, shown under a new option number. This self-comparison is the one comparison whose

Core comparison	Occurs in list
Panel A. Expected utility	
A_1 vs B_1	1
A_2 vs B_2	1
A_1 vs B_1	2
A_2 vs B_2	2
Panel B. Subjective expected utility	
A_3 vs B_3	3
A_4 vs B_4	3
A_3 vs B_3	4
A_4 vs B_4	4
Panel C. Transitivity	
X vs Y	5
X vs Z_1	5, 6
X vs Z_2	6
X vs Z_3	5, 6
Y vs Z_1	5
Y vs Z_2	7
Y vs Z_3	5, 7

Table 4: Core comparisons

truthful answer is known in advance. Therefore, it can serves as a test of whether subjects reports correctly when their preference is known, which is similar to an induced value test in experiments.

After the final list, subjects answer two open-ended questions about particular comparisons they made earlier. They are asked to describe their thinking process of decision-making. Where a subject's answers allow it, the two questions cover both kinds of answer. The first is drawn at random from the core comparisons the subject answered with a ranking or a tie, and the second from those they reported they could not rank. Under *Multi-list ABC* the second pool is empty by construction, so both questions are drawn at random from the first. The same holds under *Multi-list AMC-1* and *Multi-list AMC-2* for a subject who never reported that they could not rank a pair.

6.2 Implementation

In the experiment, subjects read instructions for the elicitation mechanisms, work through a short practice list, and then take a comprehension quiz to confirm their understanding. The quiz must be answered correctly in full before the experiment proceeds; A bonus of $2.00 is

	Plain	Verification	Total
ABC	53	91	144
AMC-1	53	95	148
AMC-2	50	80	130
Total	156	266	422

Table 5: Number of Subjects in Each Arm

paid for getting the whole quiz right on the first attempt.

The experiment is pre-registered at AsPredicted in September, 2026. The experiment was implemented at Toronto Experimental Economics Lab (**TEEL**), with subjects recruited using ORSEE. In all 422 subjects took part, 144 under Multi-list ABC, 148 under AMC-1 and 130 under AMC-2, of whom 156 were in the plain arm and 266 in the verification arm.

7 Results

The plain and verification arms face an identical experiment up to the final round. For each of the 15 core comparisons, I compare the distribution of answers across the two arms with a Fisher exact test within each mechanism. Out of the 45 tests, 5 are significant at the 10 percent level, 3 at 5 percent, and 1 at 1 percent. Applying a Holm correction for the family-wise error rate across all 45 tests, the smallest adjusted p-value is 0.254. I therefore pool the two arms within each mechanism throughout, except where the outcome is consistency with the verification round.

The results will pool the *Multi-list AMC-1* and *Multi-list AMC-2* together and then compare the difference between the two closely at the end.

7.1 Prevalence of Reported Incomplete Preference

If a subject reports a strict ranking or a tie, call it a complete preference report. If a subject reports "cannot rank", call it a incomplete preference report. Incomplete preference is reported by a minority of subjects (Table 6). Of the 278 subjects in the *Multi-list AMC* mechanisms, 60 (21.6 %) report incomplete preference in at least one core comparisons. At the comparison level, the proportion of incomplete preference is 3.1% across all the core comparisons. Therefore, incomplete preference is not reported widely.

	At least one *I do not know* (% of subjects)	Share of core comparisons answered *I do not know* (%)
AMC	21.6	3.1

Notes: AMC-1 and AMC-2 pooled. Core comparisons only, and only those the subject answered freely; comparisons settled by the carry-on rule are excluded. Column 1: 60 of 278 subjects. Column 2: 115 of 3672 comparisons. ABC is not shown: it offers no *I do not know* answer, so both rates are zero there by construction.

Table 6: Prevalence of reported incomplete preference

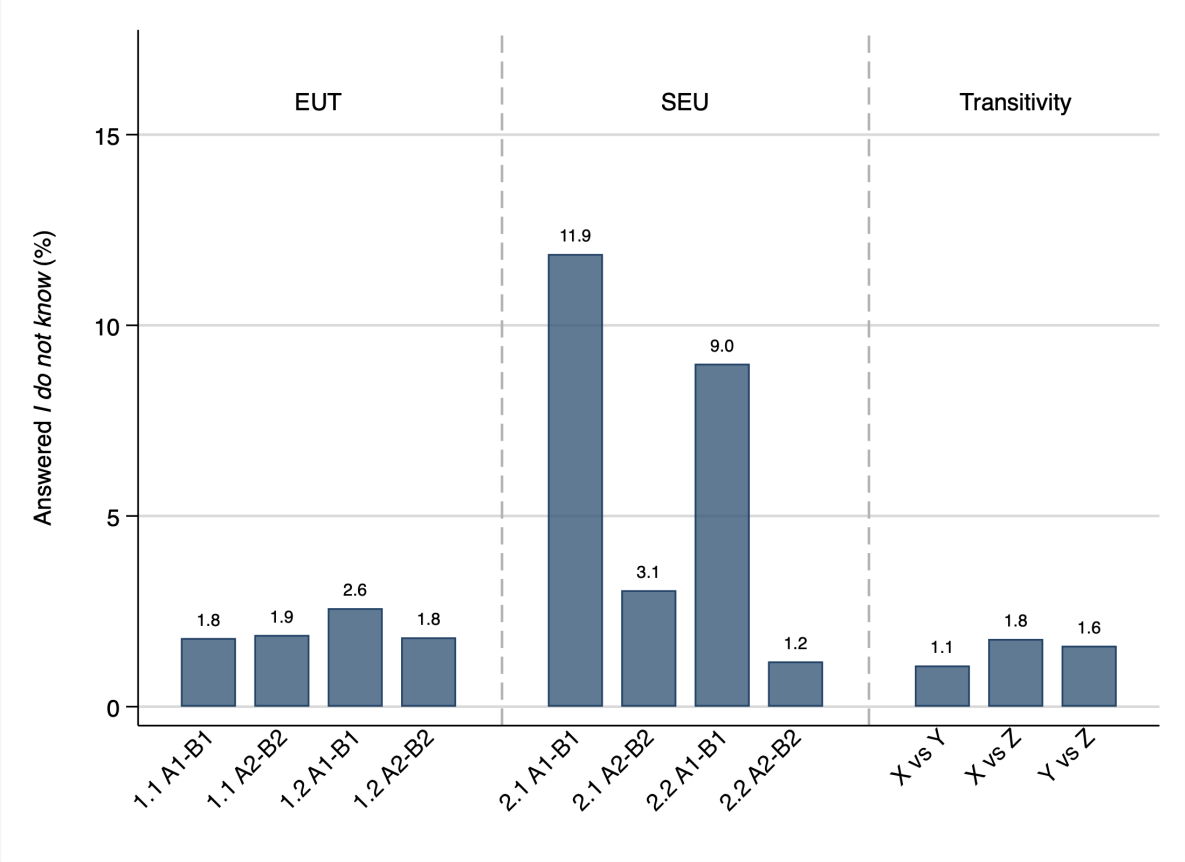
Figure 1 reports the proportion of incomplete preference report for each core comparisons[10], and it is not spread evenly. Grouping the comparisions by the test they feed, the rate is 2.0% across the four expected-utility comparisons and 1.6 % across the three transitivity comparisons, but 6.4% across the four comparisons drawn from the ambiguous urn, a gap of 4.7 percentage points against the other two groups together ($p < 0.001$, standard errors clustered on the subject). Four comparisons thus account for three-fifths of all the incomplete preference reported on core comparisons.

Within the ambiguity group the reports concentrate on the A_1 versus B_1 comparison. A_1 bets on red, which wins with known probability 1/3, and B_1 bets on blue, whose probability is unknown and lies between 0 and 2/3. That comparison has 11.9 % of incomplete preference in list 3 and 9.0 % in list 4, against 3.1% and 1.2% for the A_2 versus B_2 comparisons that follow, where A_2 bets on red or yellow with unknown probability between 1/3 and 1 and B_2 on blue or yellow, so that adding yellow turns B_2 into a known 2/3. The two comparisons are similar in format: in each of them, one option carries a known probability at the midpoint of the other option's range that is 2/3 wide. What separates them is the level. A known 1/3 against $[0, 2/3]$ leads about five times as often as a known 2/3 against $[1/3, 1]$. Therefore, a 1/3 probability versus $[0, 2/3]$ probability is a harder comparison than 2/3 versus $[1/3, 1]$.

7.2 Behavioural Incentive Compatibility

The *Multi-list AMC-1* and *Multi-list AMC-2* are b-RCSIC theoretically. In the experiment, we test the behavioural incentive compatibility.

[10] X versus Z and Y versus Z, group the three Z options together, since these are the same bet at sure payments of \$0.00, \$0.20 and \$0.50; those two bars therefore rest on 732 and 501 comparisons drawn from the same 278 subjects, roughly 2.6 and 1.8 apiece, against about 278 for each of the other nine bars.



Notes: AMC-1 and AMC-2 pooled, comparison level. Bound comparisons are excluded, so each bar is over the comparisons subjects answered freely. Denominators: 1.1 A1-B1 278; 1.1 A2-B2 266; 1.2 A1-B1 271; 1.2 A2-B2 275; 2.1 A1-B1 278; 2.1 A2-B2 262; 2.2 A1-B1 278; 2.2 A2-B2 253; X vs Y 278; X vs Z 732; Y vs Z 501. The 11 main comparisons are grouped by the test they feed: EUT (lists 1.1 and 1.2), SEU (lists 2.1 and 2.2) and transitivity (X, Y, Z).

Figure 1: Reported incomplete preference by core comparison

Share answering (%)		
Ranked one above the other	Exactly same	Cannot rank
7.6	91.0	1.4

Notes: AMC-1 and AMC-2 pooled. The self-comparison puts an option against a copy of itself, so *exactly same* is the only true answer. One comparison per subject; denominator 278. The row sums to 100.

Table 7: The self-comparison

	Share answering (%)			
	Dominant option	Dominated option	Exactly same	Cannot rank
ABC	98.5	1.5	–	–
AMC	97.9	0.7	0.7	0.6

Notes: Comparison level, dominance comparisons the subject answered freely. A dominance comparison is one where one option first-order stochastically dominates the other; this includes the pairs against the monotonicity probes C, D, E and F. Denominators: ABC 1093, AMC 2143. Each row sums to 100. ABC offers only the two rankings, so the last two cells do not apply.

Table 8: Dominance comparisons

First of all, we can examine if subjects tell the truth when their preference is known. Each subject faces one self comparison, in which the two options are identical. In the self comparison, their preference is indifference. A correct answer is to answer "I rank the two options exactly the same". The caveat noted above applies here. Under AMC mechanisms, an indifferent subject bears no cost from reporting a strict ranking instead. So we should expect some subjects to answer a strict ranking instead of a tie. But a report of "I cannot rank the two options" is definitely a wrong answer. Each subject also faces dominance comparisons, in which one option *FOSD* the other. In a dominance comparison, if they respect *FOSD*, they preference is a strict preference for the dominant choices, so the correct answer ranks it above the other.

Table 7 shows the results of the self-comparison. 91.0 % answered that they rank the two options exactly the same, 7.6 % ranked one above the other, and 1.4 % reported that they did not know how to rank them. The middle group is what the caveat predicts rather than a failure: an indifferent subject under AMC mechanisms loses nothing by reporting a strict ranking. Only the last group is a definite mistake, and at 1.4 % it is small.

Table 8 turns to the dominance comparisons at the comparison level. Subjects take the dominating option in 98.5 % under ABC and in 97.9% under AMC, and the difference is not significant ($p = 0.480$, with standard errors clustered on the subject). Therefore, in both self comparisons and dominance comparisons, most subjects report their preference truthfully.

	Consistent with the earlier answer (%)
ABC	71.4
AMC	83.3

Notes: Verification-arm subjects only. The verification task repeats one core comparison the subject already answered; consistent means the same answer, with a tie counted as consistent. Subjects whose verified comparison was answered *cannot rank* are excluded. Denominators: ABC 91, AMC 168. Two-sample proportion test, p = 0.024. All subjects.

Table 9: Consistency with the verification round

Secondly, I examine whether the report in the mechanisim is consistent with the choice they made in the verification round when a subject reported a complete preference, including a strict ranking or a tie.

The verification round repeats one core comparison the subject answered and offers a straight choice between its two options ad the option chosen is their final payoff. Of the 266 verification-arm subjects, 7 had reported that they could not rank the verified pair and are set aside, leaving 259: 91 under ABC and 168 under AMC. Table 9 gives the share whose verification choice matches their earlier report. It is 71.4 percent under ABC and 83.3 percent under AMC, a difference of 11.9 percentage points ($p = 0.024$, two-sample proportion test). Therefore, for most subjects, their report of complete preference is consistent with their truthful preference and the consistency is higher when they are given the option of "cannot rank". A large majority of reported complete preferences are confirmed in the verification round, and they are confirmed more often in the AMC mechanisms.

Third, I examine whether reports of incomplete preference are truthful. Direct evidence is not available as an incomplete preference cannot be induced. I therefore turn to two kinds of indirect evidence: the time subjects took to answer each comparison, and how difficult they said they found it, as coded from their exit-survey answers.

Subjects take longer over the comparisons they cannot rank. Restricting to the AMC mechanisms, the mean time to answer is 17.4 seconds if subjects report a complete preference, and 28.8 seconds if subjects report "cannot rank" for the 115 reported, a difference of 11.3 seconds ($p < 0.001$). The gap is not driven by the outlier: the medians are 13.1 and 24.8 seconds. Longer time to answer is consistent with the explanation that subjects find the comparisons harder when they report an incomplete preference.

Each subject in AMC mechanisms answered two open-ended questions, one about a comparison they report a complete preference, one about a comparison they had reported incomplete

	How hard the subject found it (%)		
The comparison was answered	Easy	Moderate	Difficult
With a complete preference	64.4	30.3	5.3
Cannot rank	46.7	31.7	21.7

Notes: AMC-1 and AMC-2 pooled. The unit is the exit-survey answer; each subject gives two, each about one comparison they had already made. Rows group those answers by what that comparison was answered: a complete preference means a strict ranking or *exactly the same*. Each row sums to 100. Denominators: complete 492, cannot rank 60. 4 answers coded *not informative* are dropped. Two-sample proportion tests by column, p = 0.007, 0.826, 0.000. All subjects.

Table 10: Difficulty of the decision described in the exit survey

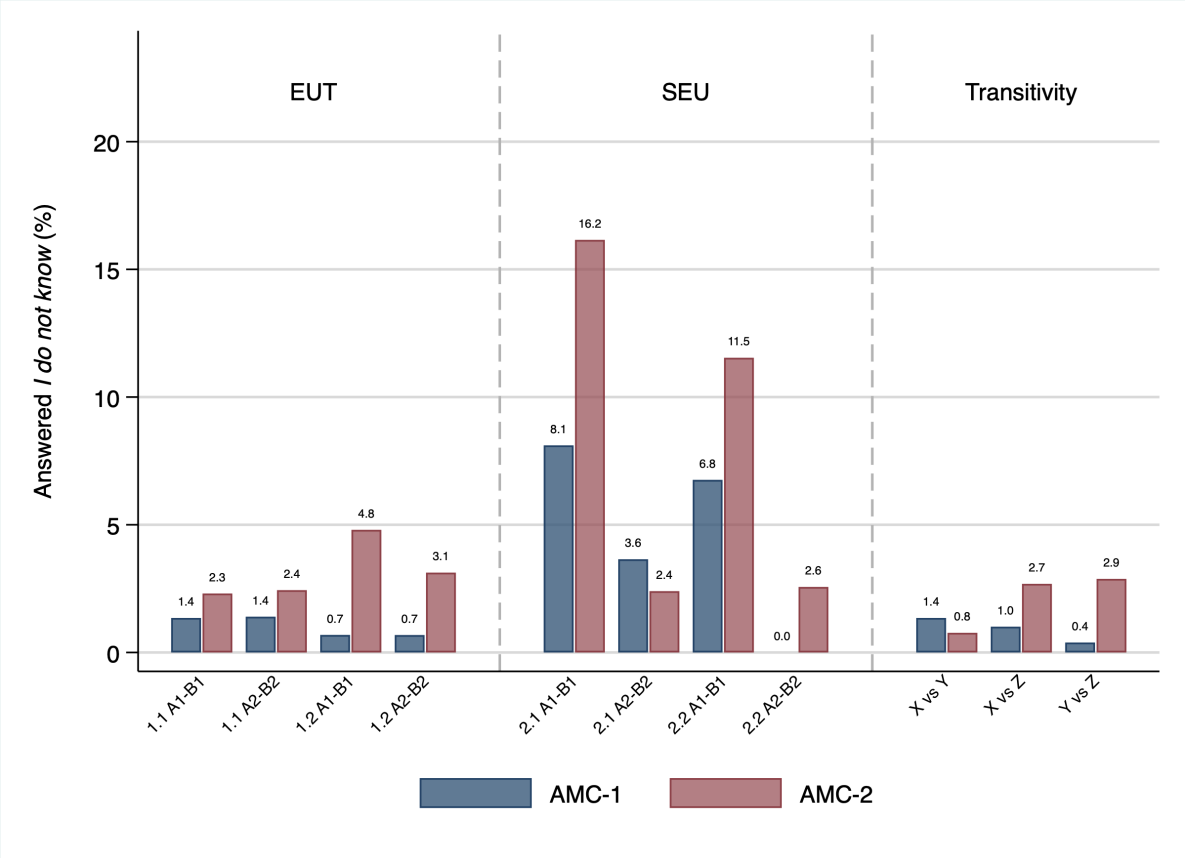
preference if applicable. I classified each answer by the difficulty level based on their description in the survey, using Claude Opus 5. The model was instructed to conduct an inductive text analysis and to categorize how difficult the decision was, based on what they wrote about their decision-making experience. The model is given the text only and no other information. The four categories it returned were easy or clear, moderate or weighing a trade-off, difficult or explicitly hard, and not informative. The full prompt, and examples of each category, are in Appendix.

Table 10 reports the distribution. 5.3% of the descriptions of a comparison the subject report a complete preference were coded difficult or explicitly hard; 21.7 % of the descriptions of a comparisons the subject report a incomplete preference were coded difficult or explicitly hard. The share of easy/clear category moves the other way, 64.4% among complete preference against 46.7 % among incomplete preference. Both differences are significant ($p = 0.007$ for easy and $p < 0.001$ for difficult), while the moderate share is unchanged ($p = 0.826$). Four answers coded not informative are excluded. Both time and survey point the same way: the comparisons with incomplete preference are the ones they spend longer time to answer and describe as harder decisions.

Overall, there is evidence for the incentive compatibility of the *Multi-list AMC* mechanisms on both complete preference and incomplete preference.

7.3 Compare *Multi-list AMC-1* and *Multi-list AMC-2*

Both *Multi-list AMC-1* and *Multi-list AMC-2* are *b-RCSIC*, which means the incentive they place on the incomplete preference is only weak: hiding their incomplete preference and report a ranking instead is not always penalized. But they differ in the path after a incomplete preference is reported. Under *Multi-list AMC-1*, reporting that the held option



Notes: Comparison level, AMC-1 and AMC-2 shown separately. Bound comparisons are excluded. Denominators, AMC-1/AMC-2: 1.1 A1-B1 148/130; 1.1 A2-B2 143/123; 1.2 A1-B1 146/125; 1.2 A2-B2 147/128; 2.1 A1-B1 148/130; 2.1 A2-B2 137/125; 2.2 A1-B1 148/130; 2.2 A2-B2 136/117; X vs Y 148/130; X vs Z 397/335; Y vs Z 258/243. The 11 main comparisons are grouped by the test they feed: EUT (lists 1.1 and 1.2), SEU (lists 2.1 and 2.2) and transitivity (X, Y, Z). All subjects.

Figure 2: Reported incomplete preference by core comparison, *Multi-list AMC-1* and *Multi-list AMC-2*

and the new option cannot be ranked leaves the subject holding what they already held, which is exactly what reporting that the held option ranks above the new one does. Under *Multi-list AMC-2*, the new option may be carried on if no options in the buffer set W are better. I investigate if they perform differently in the lab. The most prominent distinction is the proportion of incomplete preference report.

Under *Multi-list AMC-1*, 16.2 % of subjects report incomparability at least once, lower than 27.7 percent under *Multi-list AMC-2* ($p = 0.028$ for Fisher exact test), and the rate of reported incomplete preference is 2.0% against 4.4% percent at the comparison-level ($p = 0.010$). Figure 2 shows that this gap is not the work of one comparison: *Multi-list AMC-2* is the higher of the two in nine of the eleven bars.

One explanation of this gap is the easiness of hiding incomplete preference safely without leading to a non-maximal outcome. In both mechanisms, given the currently holding option

	Share answering (%)			
	Old above new	New above old	Exactly same	Cannot rank
AMC-1	68.6	18.6	5.4	7.4
AMC-2	58.5	21.5	6.2	13.8

Notes: The first core comparison of lists 2.1 and 2.2, the two urn-6 lists, pooled. Both open with the subject holding A1 while B1 arrives as the challenger, so *old* is A1 and *new* is B1 in each. Comparison level; a subject contributes up to two. Denominators: AMC-1 296, AMC-2 260. Each row sums to 100. Two-sample proportion tests, AMC-1 against AMC-2, by column: p = 0.013, 0.384, 0.705, 0.014. All subjects.

Table 11: The opening comparison of lists 3 and 4, *Multi-list AMC-1* and *Multi-list AMC-2*

c_{i-1} and the new option l_i, if a subject report that they rank c_{i-1} above l_i whenever she can not compare the two options, she can still get a maximal outcome in the end. The reverse does not hold: a subject who reports that l_i ranks above c_{i-1} in the same circumstances may not. An example makes the gap concrete. Let $l_0 \succ^+ l_2$, $l_0 \bowtie^+ l_1$ and $l_1 \bowtie^+ l_2$, with the subject starting from l_0. A subject who always reports c_{i-1} is ranked above l_i whenever $c_{i-1} \bowtie^+ l_i$ and report truthfully elsewhere keeps l_0 throughout and is paid l_0 in both mechanisms, which is maximal. A subject who always reports l_i is ranked above c_{i-1} whenever $c_{i-1} \bowtie^+ l_i$ takes l_1 and then l_2, and is paid l_2 even though $l_0 \succ^+ l_2$.

Given that in *Multi-list AMC-1*, the report of c_{i-1} is ranked above l_i and c_{i-1} and l_i cannot be ranked carry forward the same option to the next round, hiding incomplete preference by reporting c_{i-1} is ranked above l_i is more salient in *Multi-list AMC-1* than in *Multi-list AMC-2*, therefore, the reported incomplete preference rate is lower in *Multi-list AMC-1*. This explanation can be supported by further evidence. In the lists 3 and 4, where the reported incomplete preference is concentrated, all the subject holds A_3 and meets B_3 in the first round in both arms. As shown in Table 11, subjects under *Multi-list AMC-1* rank the held option above the new one 68.6 percent of the time, v.s. 58.5 percent under *Multi-list AMC-2* ($p = 0.013$), and the percentage of reported incomplete preference is 7.4 % versus 13.8 % ($p = 0.014$). The share ranking the new option above the held one is statistically unchanged, 18.6 % v.s. 21.5 % ($p = 0.384$), as is the share reporting a tie, 5.4 % v.s. 6.2 % ($p = 0.705$). The increase of reported incomplete preference in *Multi-list AMC-2* comes from the decreases of the report of the held option above the new option.

In addition, the two mechanisms perform similarly in terms of Behavioural Incentive Compatibility. On the self-comparisons, 89.9 % of *Multi-list AMC-1* subjects and 92.3 % of *Multi-list AMC-2* subjects choose "I rank the two options exactly the same" (Fisher exact $p = 0.533$). The two mechanisms are indistinguishable in the dominance comparisons, each

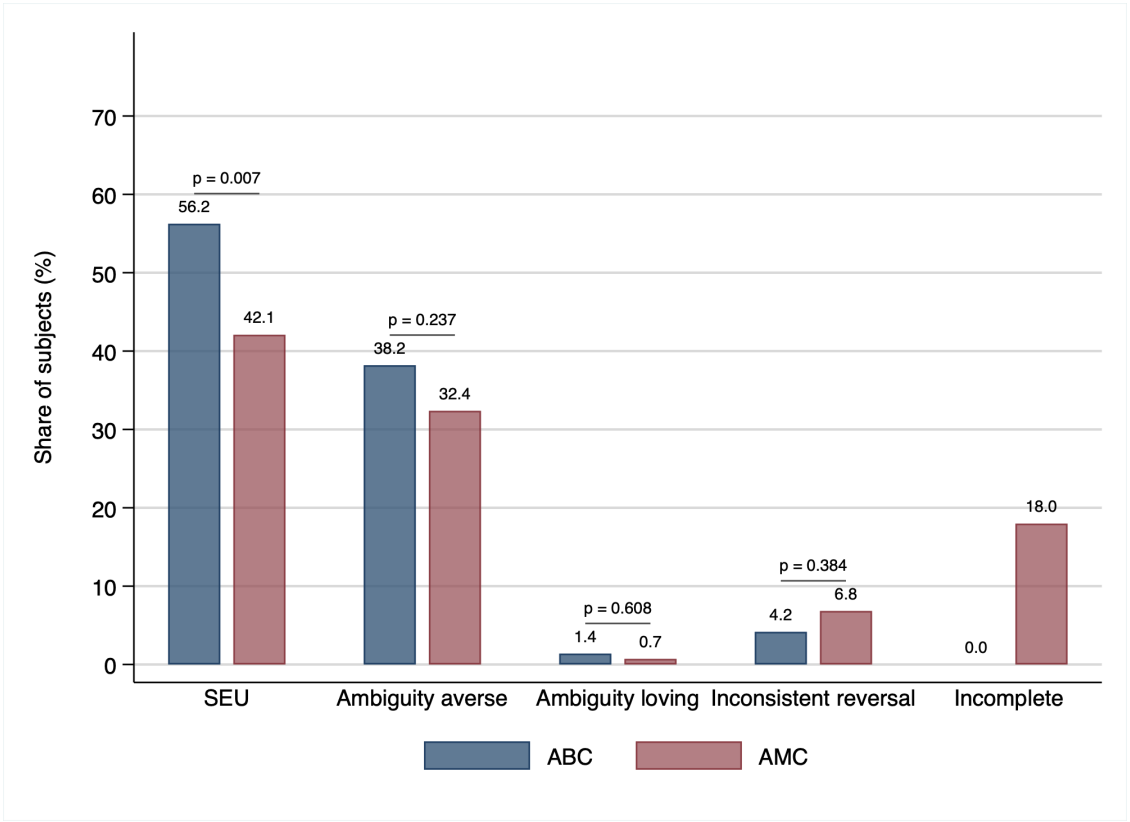
taking the dominating option in 97.9 % ($p = 0.987$). Consistency with the verification round is 85.9 % under *Multi-list AMC-1* and 80.3 % under *Multi-list AMC-2* ($p = 0.407$). The two indirect evidence on the report of incomplete preference behave the same way in each arm as well. Comparisons with reported incomplete preference take longer than those given a complete preference under both mechanisms, 31.3 seconds v.s. 17.3 seconds in *Multi-list AMC-1* and 27.4 seconds v.s. 17.6 seconds in *Multi-list AMC-2* (both $p < 0.001$), and the survey answers written about them are coded difficult or explicitly hard far more often, 25.0 % against 6.7 % under *Multi-list AMC-1* ($p = 0.002$) and 19.4 % against 3.6 % under AMC-2 ($p < 0.001$). Therefore, the two mechanisms differ significantly in the proportion of elicited incomplete preference, not in the performance in incentive compatibility.

7.4 Incomplete Preference and EUT, SEU, and Transitivity

The 7 lists already have the test of **EUT**, **SEU**, and transitivity built in. I examine how allowing the report of incomplete preference impact the violation of **EUT**, **SEU**, and transitivity.

Given that incomplete preference is concentrated in the ambiguity group, I examine the relationship between incomplete preference and **SEU**. The relationship between incomplete preference and **EUT** and transitivity are in the appendix.

Figure 3 gives the distribution of subjects across SEU classes: Ambiguity Natural, Ambiguity Averse, Ambiguity Seeking, Non-monotonic, and Incomplete preference. Under *Multi-list AMC*, 18.0 % subject report incomplete preference in at least one core comparisons in either list 3 and 4 and be classified as the incomplete preference class. Non-monotonic means their choices in list 3 and list 4 have preference reversals that violate monotonicity. This incomplete preference class is empty in *Multi-list ABC* by construction, and therefore, the rate of other classes are correspondingly lower under *Multi-list AMC* compared with *Multi-list ABC*. 42.1% satisfy SEU against 56.2 % in ABC ($p = 0.007$), 32.4 % are ambiguity averse against 38.2 ($p = 0.237$), and 0.7 percent ambiguity loving against 1.4 ($p = 0.608$). The one class that does not fall is the inconsistent reversal, at 6.8 against 4.2 percent ($p = 0.384$). Allowing incomplete preference therefore reduces both the measured consistency with SEU and the measured violations of it. As Table 12 shows, among the subjects classified as satisfying SEU, ambiguity averse or ambiguity loving, 58.7 % in *Multi-list ABC* satisfy SEU v.s. 56.0 % under *Multi-list AMC*, and the difference is far from significant (Fisher exact $p = 0.658$). Therefore, the report of incomplete preference reduces SEU consistency and SEU violations evenly.



Notes: Subject level, one observation per subject. AMC-1 and AMC-2 pooled. Bars give the share of subjects in each SEU class. Denominators: ABC 144, AMC 278. Fisher exact test of each class against the rest. *Incomplete* cannot arise in ABC, which offers no *cannot rank* answer, so it is not tested. All subjects.

Figure 3: SEU classification

	Satisfies SEU (%)	Ambiguity averse or loving (%)
ABC	58.7	41.3
AMC	56.0	44.0

Notes: Subject level, AMC-1 and AMC-2 pooled. Restricted to subjects classified as SEU, ambiguity averse and ambiguity loving; all other classes are excluded. Denominators: ABC 138 (81 / 57), AMC 209 (117 / 92). Each row sums to 100. Fisher exact test, p = 0.658. All subjects.

Table 12: SEU and its named violations

This evidence can shed light on the question of how do subjects with incomplete preference make choices when they are forced to. There is thus no evidence of systematic behaviour of subjects with incomplete preference. The reports remove subjects from both sides of the SEU classification in much the same proportion, which is what one would expect if the answers these subjects would give under a forced ranking is close to stochastic choice and carry no systematic information about ambiguity attitude.

8 Conclusion

This paper establishes three results on experiments with incomplete preferences. First, it characterizes the frontier of incentive compatibility: truth-telling can be strictly incentivized on the comparable side, but only weakly on the incomparable side. This frontier also determines how reported incomparability should be read. A subject who can rank two options never gains, and may lose, by claiming that she cannot, whereas a subject who cannot rank them may report a ranking at no cost. Reported incomparability is therefore a conservative measure, and the rates observed here, 21.6% of subjects and 3.1% of comparisons, are best read as lower bounds. Second, multi-list *AMC* mechanisms attain this frontier. Third, these mechanisms perform well in the laboratory: subjects answer correctly when the truthful answer is known, and a verification round confirms 83.3% of the rankings reported under *AMC*, against 71.4% when a ranking is forced. Thus, allowing subjects to say that they cannot rank two options is useful even for experimenters who are not interested in incompleteness itself, since it makes the rankings that subjects do report more reliable.

These results come with three limitations. The first concerns the lower bound on the thickness of incomparability, b. Incomparability has to be thick enough to be able to be separated from indifference, which is a further reason why measured incompleteness is a lower bound. Secondly, I assume transitivity throughout. Without transitivity, incentive compatibility is not well-defined, since a cycle in preferences can leave no option that is not worse than some other option. Nonetheless, transitivity is a substantive restriction. Future work can work on the relaxation of transitivity. The third limitation is about the beliefs DMs hold over future options in *Multi-list AMC* mechanisms. A subject who expects a better option to arrive later has no reason to answer the current comparison truthfully, and a subject who holds a probability distribution over the composition of the lists evaluates her payment as an act, which brings back the concerns that motivate the design. Lastly, I do not impose any behavioural assumption on incomplete preferences, such as deferral, delegation or randomization. Additional behavioural assumptions could push the frontier of incentive compatibility further,

but at a cost: it is built on behavioral premises that are themselves open to question.

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A Proofs for Section 4

A.1 Preliminaries

For acts f, g, write $f \geq_P g$ if $f(\omega) \succsim^+ g(\omega)$ for every $\omega \in \Omega$.

This relation is the extension $\succsim_0^\star$ of Section 4; the case $G = \emptyset$ of Lemma A.1 below shows that it belongs to $\mathcal{C}(\succsim^+)$. For a state ω, let $A(\phi, \omega) = \{\phi(m')(\omega) : m' \in M\}$ be the set of outcomes within reach in ω.

Lemma A.1 (Witness)**.** *Let $\succsim^+ \in \mathcal{R}$, let f be an act and let G be a set of acts. The following are equivalent:*

(a) for every $g \in G$ there is $\omega_g \in \Omega$ with $f(\omega_g) \succ^+ g(\omega_g)$;

(b) there exists $\succsim^\star \in \mathcal{C}(\succsim^+)$, which may depend on f and G, such that $f \succ^\star g$ for every $g \in G$.

In particular, for acts f, g: some $\succsim^\star \in \mathcal{C}(\succsim^+)$ has $f \succ^\star g$ if and only if $f(\omega) \succ^+ g(\omega)$ for some $\omega \in \Omega$.

Proof. (b)⇒(a) is Assumption 11(ii).

(a)⇒(b). Define $h \succsim^\star_{f,G} h'$ if either (1) $h \geq_P h'$, or (2) $h \geq_P f$ and $g \geq_P h'$ for some $g \in G$. Type (2) is what transitivity forces once f is ranked weakly above each $g \in G$: if $h \geq_P f$ and $g \geq_P h'$, Assumption 11(i) gives $h \succsim^\star f$ and $g \succsim^\star h'$, so $f \succsim^\star g$ forces $h \succsim^\star h'$.

The proof checks that adding the pairs of type (2) breaks none of the assumptions. The key fact is that each such pair comes with a state in which the first act is strictly better. If

(h, h') is of type (2) through g, then by transitivity of $\succsim^+$

$$h(\omega_g) \succsim^+ f(\omega_g) \succ^+ g(\omega_g) \succsim^+ h'(\omega_g), \qquad \text{so} \qquad h(\omega_g) \succ^+ h'(\omega_g). \tag{$*$}$$

Reflexivity follows from (1).

Transitivity. Let $h_1 \succsim^\star_{f,G} h_2 \succsim^\star_{f,G} h_3$. If both pairs are of type (1), then $h_1 \geq_P h_3$. Otherwise the first act is $\geq_P f$ (directly, or through $h_1 \geq_P h_2 \geq_P f$) and the last is $\leq_P g$ for some $g \in G$ (directly, or through $h_3 \leq_P h_2 \leq_P g$), so (h_1, h_3) is of type (2).

Agreement on constant acts. Identify each outcome with the constant act that pays it in every state. If $c \succsim^+ d$, then $c \geq_P d$, so $c \succsim^\star_{f,G} d$ by type (1). Conversely, suppose $c \succsim^\star_{f,G} d$. If the pair is of type (1), then $c \geq_P d$, which for constant acts says $c \succsim^+ d$. If it is of type (2), then $(*)$ gives $c \succ^+ d$, since a constant act takes the same value in every state, including ω_g; so again $c \succsim^+ d$. Thus type (2) never ranks two sure outcomes that $\succsim^+$ does not already rank.

State Monotonicity (i). If $h \geq_P h'$, then $h \succsim^\star_{f,G} h'$ by type (1). Suppose in addition that $h(\omega_1) \succ^+ h'(\omega_1)$ for some ω_1; we show that $h' \succsim^\star_{f,G} h$ fails, so that $h \succ^\star_{f,G} h'$. A pair (h', h) of type (1) would mean $h' \geq_P h$, so $h'(\omega_1) \succsim^+ h(\omega_1)$, contradicting $h(\omega_1) \succ^+ h'(\omega_1)$. A pair (h', h) of type (2) would, by $(*)$, have a state ω_g with $h'(\omega_g) \succ^+ h(\omega_g)$, contradicting $h \geq_P h'$. Thus type (2) never reverses a strict statewise dominance.

State Monotonicity (ii). Suppose $h \succ^\star_{f,G} h'$, that is, $h \succsim^\star_{f,G} h'$ and not $h' \succsim^\star_{f,G} h$. If (h, h') is of type (2), $(*)$ gives a state in which h is strictly better. Otherwise (h, h') is of type (1), so $h(\omega) \succsim^+ h'(\omega)$ for every ω. Since $h' \succsim^\star_{f,G} h$ fails, in particular $h' \geq_P h$ fails, so some ω has $h'(\omega) \not\succsim^+ h(\omega)$. At that ω, $h(\omega) \succsim^+ h'(\omega)$ and $h'(\omega) \not\succsim^+ h(\omega)$, that is, $h(\omega) \succ^+ h'(\omega)$.

Conclusion. Fix $g \in G$. Since $f \geq_P f$ and $g \geq_P g$, the pair (f, g) is of type (2), so $f \succsim^\star_{f,G} g$. For $g \succsim^\star_{f,G} f$, type (1) would need $g \geq_P f$, and so would type (2), whose first act must be statewise at least as good as f; both fail at ω_g, where $f(\omega_g) \succ^+ g(\omega_g)$. Hence $f \succ^\star_{f,G} g$ for every $g \in G$, and $\succsim^\star_{f,G}$ is the extension required in (b). If $G = \emptyset$, there are no pairs of type (2), so $\succsim^\star_{f,G}$ is $\geq_P$ itself; the checks above then show that $\geq_P \in \mathcal{C}(\succsim^+)$, so $\mathcal{C}(\succsim^+) \neq \emptyset$. □

Lemma A.2. *Let $\succsim^+ \in \mathcal{R}$, let $m^\star \in \bar{M}(\phi, \succsim^+)$, and let $m' \in M$ satisfy $\phi(m^\star)(\omega_0) \succ^+ \phi(m')(\omega_0)$ for some $\omega_0 \in \Omega$. Then $\phi(m') \not\succsim^\star \phi(m^\star)$ $\forall$ $\succsim^\star \in \mathcal{C}(\succsim^+)$. In particular, $\forall$ $\succsim^\star \in \mathcal{C}(\succsim^+)$ that can compare $\phi(m^\star)$ and $\phi(m')$, $\phi(m^\star) \succ^\star \phi(m')$.*

Proof. Write $t = \phi(m^\star)$ and $g = \phi(m')$. Since $m^\star \in \bar{M}(\phi, \succsim^+)$, no message pays strictly

more than $m^\star$ in any state; in particular

$$g(\omega) \not\succ^+ t(\omega) \qquad \forall \omega \in \Omega. \tag{$\dagger$}$$

Also $t(\omega_0) \succ^+ g(\omega_0)$ by assumption. Fix $\succsim^\star \in \mathcal{C}(\succsim^+)$ and suppose, for contradiction, that $g \succsim^\star t$.

Step 1: if $g \succsim^\star t$, then $g(\omega)$ and $t(\omega)$ are comparable under $\succsim^+$, $\forall \omega \in \Omega$. Suppose instead that $g(\omega_1) = (a, \alpha) \bowtie^+ t(\omega_1)$ for some ω_1. We add a small amount of money to g in state ω_1 and show that the resulting act would have to be strictly better than t without being strictly better than t in any state.

- By Assumption 7, the set $\{\alpha' : (a, \alpha') \bowtie^+ t(\omega_1)\}$ is open and contains α, so $\exists \varepsilon > 0$ with $(a, \alpha + \varepsilon) \bowtie^+ t(\omega_1)$.
- Let k be the act that equals g except that $k(\omega_1) = (a, \alpha + \varepsilon)$. Then $k(\omega) = g(\omega)$ for $\omega \neq \omega_1$, and $k(\omega_1) \succ^+ g(\omega_1)$ by Assumption 6. By Assumption 11(i), $k \succ^\star g$.
- Since $k \succ^\star g$ and $g \succsim^\star t$, we get $k \succ^\star t$ by transitivity (Assumption 8).
- By Assumption 11(ii), $k \succ^\star t$ requires a state ω_2 with $k(\omega_2) \succ^+ t(\omega_2)$. There is none. At ω_1, $k(\omega_1) \bowtie^+ t(\omega_1)$ by the choice of ε. At $\omega_2 \neq \omega_1$, $k(\omega_2) = g(\omega_2)$, which is not strictly better than $t(\omega_2)$ by ($\dagger$).

This contradiction shows that $g(\omega)$ and $t(\omega)$ are comparable, $\forall \omega \in \Omega$.

Step 2: Fix $\omega \in \Omega$. By Step 1, $g(\omega)$ and $t(\omega)$ are comparable, and by ($\dagger$), $g(\omega)$ is not strictly better; hence $t(\omega) \succsim^+ g(\omega)$. So $t(\omega) \succsim^+ g(\omega)$ $\forall \omega \in \Omega$, and $t(\omega_0) \succ^+ g(\omega_0)$. By Assumption 11(i), $t \succ^\star g$, which contradicts $g \succsim^\star t$. Hence $g \not\succsim^\star t$.

To conclude, if $\succsim^\star$ can compare t and g, then $t \succsim^\star g$ or $g \succsim^\star t$; since $g \not\succsim^\star t$, we have $t \succsim^\star g$ and not $g \succsim^\star t$, that is, $t \succ^\star g$. □

Lemma A.3. *A mechanism ϕ satisfies b-RCSIC if and only if, $\forall \succsim^+ \in \mathcal{P}_b$:*

(a) $T(\succsim^+) \subseteq \bar{M}(\phi, \succsim^+)$; and

(b) $\forall m'$ that contains a comparable-side lie and $\forall m^\star \in T(\succsim^+)$, $\exists \omega \in \Omega$ with $\phi(m^\star)(\omega) \succ^+ \phi(m')(\omega)$.

Proof. The proof of Lemma 2 treats each $\succsim^+$ separately. Restricting it to $\succsim^+ \in \mathcal{P}_b$ gives the result. □

A.2 Proofs of the Results in Section 4

Lemma A.4 (An indifferent DM cannot be penalized for declaring a strict ranking)**.** *Let Ω be finite, and replace each M_i by $M_i^{\sim} = \{d_{1i} \hat{\succ} d_{2i},\ d_{2i} \hat{\succ} d_{1i},\ d_{1i} \hat{\sim} d_{2i}\}$, where the truthful report declares the strict preference when there is one and $\hat{\sim}$ under indifference; the message space is $\prod_i M_i^{\sim}$. Let ϕ be a mechanism on this message space under which truthful reports are optimal for complete preferences: $\phi(m) \succsim^{\star} \phi(m')$ for every m' in this message space, every complete $\succsim^{+} \in \mathcal{R}$, every $m \in T(\succsim^{+})$ and every extension $\succsim^{\star}$ of $\succsim^{+}$ satisfying Assumptions 8–12.*

Let $\succsim_1^{+} \in \mathcal{R}$ be complete, $m^1 \in T(\succsim_1^{+})$ and p an option in D, and obtain m^2 from m^1 by changing every report $p \hat{\sim} q$ with $q \neq p$ to $p \hat{\succ} q$. Then $\phi(m^1) \not\succsim_1^{\star} \phi(m^2)$ for every extension $\succsim_1^{\star}$ of $\succsim_1^{+}$ that satisfies Assumptions 8–11.

Proof of Lemma A.4. Let $C = \{s \in X \times \mathbb{R} : s \sim_1^{+} p\}$.

Step 1: a refinement. Define $\succsim_2^{+}$ by moving p strictly above the rest of its indifference class: $p \succ_2^{+} s$ for every $s \in C \setminus \{p\}$, and $s \succsim_2^{+} t \Leftrightarrow s \succsim_1^{+} t$ for every other pair. Then $\succsim_2^{+}$ is complete and reflexive, and

$$s \succ_1^{+} t \ \Rightarrow\ s \succ_2^{+} t, \qquad s \succsim_2^{+} t \ \Rightarrow\ s \succsim_1^{+} t. \tag{1}$$

It is transitive. Let $s \succsim_2^{+} t \succsim_2^{+} u$. By (1), $s \succsim_1^{+} u$, which gives $s \succsim_2^{+} u$ unless $u = p$ and $s \in C \setminus \{p\}$. In that case $t \succsim_2^{+} p$ rules out $t \in C \setminus \{p\}$. If $t = p$, then $s \succsim_2^{+} p$ contradicts $p \succ_2^{+} s$. Otherwise $t \succ_1^{+} p \sim_1^{+} s$, so $t \succ_2^{+} s$ by (1), contradicting $s \succsim_2^{+} t$. Monotonicity in Money holds by (1), and Thick Incomparability holds vacuously. Define $\succsim_2$ on X by $x \succsim_2 y \Leftrightarrow (x, 0) \succsim_2^{+} (y, 0)$; it is transitive and reflexive because $\succsim_2^{+}$ is, and Assumption 5 holds for $\succsim_2$ and $\succsim_2^{+}$ by construction. Since $\succsim_2^{+}$ differs from $\succsim_1^{+}$ only on pairs $\{p, s\}$ with $s \in C \setminus \{p\}$, we have $m^2 \in T(\succsim_2^{+})$.

Step 2: a strict state. Suppose $\phi(m^1) \succ_1^{\star} \phi(m^2)$ for some $\succsim_1^{\star} \in \mathcal{C}(\succsim_1^{+})$. By State Monotonicity (ii) and (1), there is ω_0 with $\phi(m^1)(\omega_0) \succ_2^{+} \phi(m^2)(\omega_0)$.

Step 3: a lexicographic extension. Enumerate Ω as $\omega_1 = \omega_0, \omega_2, \ldots, \omega_n$, and let $f \succsim_2^{\star} g$ if $f(\omega_k) \sim_2^{+} g(\omega_k)$ for all k, or if $f(\omega_k) \succ_2^{+} g(\omega_k)$ at the first k with $f(\omega_k) \not\sim_2^{+} g(\omega_k)$. This relation is complete, reflexive and transitive, and it agrees with $\succsim_2^{+}$ on constant acts. It satisfies State Monotonicity: if $f(\omega) \succsim_2^{+} g(\omega)$ for all ω, the first non-indifferent state favors f; and if $f \succ_2^{\star} g$, that state has $f(\omega) \succ_2^{+} g(\omega)$. So $\succsim_2^{\star}$ is an extension of $\succsim_2^{+}$ satisfy all the needed assumptions required by this lemma, and $\phi(m^1) \succ_2^{\star} \phi(m^2)$ since the two acts are strictly ranked at ω_1. Hence the truthful report $m^2 \in T(\succsim_2^{+})$ does not attain a $\succsim_2^{\star}$-maximum,

a contradiction. □

Proof of Lemma 1. *"if".* Suppose $m \in \bar{M}(\phi, \succsim^+)$, and suppose $\phi(m') \succ^\star \phi(m)$ for some $m' \in M$ and some $\succsim^\star \in \mathcal{C}(\succsim^+)$. Assumption 11(ii) gives a state ω with $\phi(m')(\omega) \succ^+ \phi(m)(\omega)$, which contradicts $m \in \bar{M}(\phi, \succsim^+)$. Hence $\phi(m') \not\succ^\star \phi(m)$ for every $m' \in M$ and every $\succsim^\star \in \mathcal{C}(\succsim^+)$.

"only if". Suppose $m \notin \bar{M}(\phi, \succsim^+)$. Then some $m' \in M$ and $\omega \in \Omega$ satisfy $\phi(m')(\omega) \succ^+ \phi(m)(\omega)$. By Lemma A.1, applied with $f = \phi(m')$ and $G = \{\phi(m)\}$, $\exists \succsim^\star \in \mathcal{C}(\succsim^+)$ with $\phi(m') \succ^\star \phi(m)$, so the condition in the lemma fails for this m' and $\succsim^\star$.

□

Proof of Lemma 2. By Definition 3, ϕ satisfies CSIC if and only if, $\forall \succsim^+ \in \mathcal{R}$, conditions 1 and 2 hold for every $m^\star \in T(\succsim^+)$. Fix such a $\succsim^+$ and write $\mathcal{C} = \mathcal{C}(\succsim^+)$. It suffices to show that, for this $\succsim^+$, conditions 1 and 2 hold for every $m^\star \in T(\succsim^+)$ if and only if (a) and (b) in this lemma hold.

Condition 1 and (a). For a given $m^\star \in T(\succsim^+)$, condition 1 states that $\phi(m) \not\succ^\star \phi(m^\star)$ for every $m \in M$ and every $\succsim^\star \in \mathcal{C}$. By the first statement of Lemma 1, applied to $m = m^\star$, this holds if and only if $m^\star \in \bar{M}(\phi, \succsim^+)$. Hence condition 1 holds $\forall m^\star \in T(\succsim^+)$ if and only if $T(\succsim^+) \subseteq \bar{M}(\phi, \succsim^+)$, which is (a).

Condition 2 implies (b). Fix $m^\star \in T(\succsim^+)$ and m' that contains a comparable-side lie. By condition 2, some $\succsim^\star \in \mathcal{C}$ can compare $\phi(m^\star)$ and $\phi(m')$, and it strictly prefers $\phi(m^\star)$. Assumption 11(ii) then gives a state ω with $\phi(m^\star)(\omega) \succ^+ \phi(m')(\omega)$. Since $m^\star$ and m' were arbitrary, (b) holds.

(a) and (b) imply condition 2. Fix $m^\star \in T(\succsim^+)$ and m' that contains a comparable-side lie. By (b), some ω_0 has $\phi(m^\star)(\omega_0) \succ^+ \phi(m')(\omega_0)$.

- *Some extension can compare the two acts.* Apply Lemma A.1 with $f = \phi(m^\star)$ and $G = \{\phi(m')\}$: its condition (a) holds with $\omega_g = \omega_0$, so $\exists \succsim^\star \in \mathcal{C}$ with $\phi(m^\star) \succ^\star \phi(m')$.
- $\forall \succsim^\star \in \mathcal{C}$ *that can compare them,* $\phi(m^\star) \succ^\star \phi(m')$. By (a), $m^\star \in \bar{M}(\phi, \succsim^+)$. Together with ω_0, Lemma A.2 gives $\phi(m') \not\succsim^\star \phi(m^\star)$ $\forall \succsim^\star \in \mathcal{C}$. If $\succsim^\star$ can compare the two acts, it must therefore have $\phi(m^\star) \succsim^\star \phi(m')$ and not $\phi(m') \succsim^\star \phi(m^\star)$, that is, $\phi(m^\star) \succ^\star \phi(m')$.

Hence condition 2 holds $\forall m^\star \in T(\succsim^+)$.

Combining the three parts $\forall \succsim^+ \in \mathcal{R}$ gives the lemma. □

Proof of Proposition 1. Suppose ϕ satisfies CSIC, and suppose for contradiction that $m \in M_R$ reports $(x,0)\hat{\bowtie}(y,0)$ on a core task but $(x,0)$ and $(y,0)$ do not coexist under (ϕ, m). Pick $\succsim_0^+ \in \mathcal{R}$ with $m \in T(\succsim_0^+)$. By condition 1 and Lemma 1, $m \in \bar{M}(\phi, \succsim_0^+)$. By Assumption 6 for $\succsim_0^+$, $\forall \omega$,

$$\text{no outcome } (a,\alpha') \in A(\phi,\omega) \text{ has } \alpha' > \alpha, \text{ where } \phi(m)(\omega) = (a,\alpha). \qquad (**)$$

Coexistence sets. Let S_x be the set of states ω with $\phi(m)(\omega) = (x, \eta^-)$ for some $\eta^- \leq 0$ and $(y, \gamma^+) \in A(\phi, \omega)$ for some $\gamma^+ \geq 0$, and let $S_x^s \subseteq S_x$ consist of those states in which, in addition, $\eta^- < 0$ or such a $\gamma^+ > 0$ exists. Define S_y and S_y^s symmetrically, with $\phi(m)(\omega) = (y, \gamma^-)$, $\gamma^- \leq 0$, and $(x, \eta^+) \in A(\phi, \omega)$, $\eta^+ \geq 0$. Any pair in $S_x \times S_y$ with at least one state in $S_x^s \cup S_y^s$ is a coexistence pair. Since $(x,0)$ and $(y,0)$ do not coexist,

$$S_x^s \neq \emptyset \;\Rightarrow\; S_y = \emptyset, \qquad S_y^s \neq \emptyset \;\Rightarrow\; S_x = \emptyset.$$

Construction. Let

$$R_0 = \big\{\big((a,\alpha),(a,\alpha')\big) :\; a \in X,\; \alpha \geq \alpha'\big\},$$
$$R_{xy} = \big\{\big((x,\eta),(y,\gamma)\big) :\; \eta \geq 0 \geq \gamma\big\}, \qquad R_{yx} = \big\{\big((y,\gamma),(x,\eta)\big) :\; \gamma \geq 0 \geq \eta\big\},$$

and define $\succsim^+$ by cases:

- Case 1 ($S_x^s \neq \emptyset$): $\succsim^+ = R_0 \cup R_{xy}$, so $(x,0) \succ^+ (y,0)$.
- Case 2 ($S_y^s \neq \emptyset$): $\succsim^+ = R_0 \cup R_{yx}$, so $(y,0) \succ^+ (x,0)$.
- Case 3 ($S_x^s = S_y^s = \emptyset$): $\succsim^+ = R_0 \cup R_{xy} \cup R_{yx}$, so $(x,0) \sim^+ (y,0)$.

Cases 1 and 2 cannot both occur, since each implies that the other's set S is empty.

Step 1: $\succsim^+ \in \mathcal{R}$ and $\succsim^+ \in \mathcal{P}_b$ $\forall b$. Reflexivity follows from R_0. Transitivity: compositions within R_0 stay in R_0; composing R_0 with R_{xy} on either side stays in R_{xy}, because adding money to the x outcome or removing money from the y outcome preserves $\eta \geq 0 \geq \gamma$, and likewise for R_{yx}; two pairs of R_{xy} cannot be composed, since each starts at an x outcome and ends at a y outcome, and likewise for R_{yx}; in Case 3, $(x,\eta) \succsim^+ (y,\gamma) \succsim^+ (x,\eta')$ requires $\eta \geq 0 \geq \gamma$ and $\gamma \geq 0 \geq \eta'$, so $\eta \geq \eta'$ and the composite lies in R_0; the composition through an x outcome is symmetric. Monotonicity in money holds because outcomes of the same alternative are ranked only through R_0. Assumption 5 defines $\succsim$ as the restriction of $\succsim^+$ to zero money. Thick incomparability: outcomes of two different alternatives other than the

pair $\{x, y\}$ are never comparable, so both sets in Assumption 7 equal $\mathbb{R}$. For (x, η) and (y, γ), comparability holds exactly when $\eta \geq 0 \geq \gamma$ (Case 1), when $\gamma \geq 0 \geq \eta$ (Case 2), or when either holds (Case 3); for fixed γ the set $\{\eta : (x, \eta) \bowtie^{+} (y, \gamma)\}$ is then one of $\emptyset$, $(-\infty, 0)$, $(0, \infty)$, $\mathbb{R}$, and symmetrically for fixed η; all are open. Finally, every core pair other than $\{(x, 0), (y, 0)\}$ is incomparable at every money level, so its thickness is infinite, and $(x, 0)$, $(y, 0)$ are comparable; hence $\succsim^{+} \in \mathcal{P}_b$.

Step 2: $m \in \bar{M}(\phi, \succsim^{+})$. The strict pairs of $\succsim^{+}$ are: outcomes of the same alternative with more money; in Cases 1 and 3, $(x, \eta) \succ^{+} (y, \gamma)$ when $\eta \geq 0 \geq \gamma$, except $\eta = \gamma = 0$ in Case 3; in Cases 2 and 3, the symmetric pairs. Suppose some $(z, \beta) \in A(\phi, \omega)$ satisfies $(z, \beta) \succ^{+} \phi(m)(\omega)$. By $(**)$, (z, β) is an outcome of a different alternative from $\phi(m)(\omega)$. Since $\succsim^{+}$ compares outcomes of different alternatives only within the pair $\{x, y\}$, one of the two is an x outcome and the other a y outcome. If $\phi(m)(\omega) = (y, \gamma^{-})$ is beaten by (x, η^{+}), then $\eta^{+} \geq 0 \geq \gamma^{-}$, so $\omega \in S_y$, which is impossible in Case 1; in Case 3 the two are not both zero, so $\omega \in S_y^s$, which is also impossible; and in Case 2 no x outcome is strictly above a y outcome. The case $\phi(m)(\omega) = (x, \eta^{-})$ is symmetric. Hence no state has an available outcome strictly above $\phi(m)(\omega)$.

Step 3: contradiction. Under $\succsim^{+}$, the outcomes $(x, 0)$ and $(y, 0)$ are comparable, while m reports $(x, 0) \hat{\bowtie} (y, 0)$, so m contains a comparable-side lie under $\succsim^{+}$. For any $m^{\star} \in T(\succsim^{+})$, Step 2 implies $\phi(m^{\star})(\omega) \not\succ^{+} \phi(m)(\omega)$ $\forall \omega$. So condition (b) of Lemma 2 fails at $\succsim^{+}$, and ϕ does not satisfy CSIC, a contradiction. □

Proof of Proposition 2. Suppose ϕ satisfies CSIC and fix a core task $\{(x, 0), (y, 0)\} \in D_{core}$. Let

$$\Delta = \min\big\{|\eta - \gamma| : \ \{(x, \eta), (y, \gamma)\} \in D, \ \eta \neq \gamma\big\},$$

with $\Delta = \infty$ if the set is empty. Since D is finite, $\Delta > 0$.

Band preferences. For $\varepsilon > 0$, define $\succsim_{\varepsilon}^{+}$ on $X \times \mathbb{R}$ as follows:

- $(a, \alpha) \succsim_{\varepsilon}^{+} (a, \alpha')$ if and only if $\alpha \geq \alpha'$, for every $a \in X$;
- $(x, \eta) \succsim_{\varepsilon}^{+} (y, \gamma)$ if and only if $\eta \geq \gamma + \varepsilon$, and $(y, \gamma) \succsim_{\varepsilon}^{+} (x, \eta)$ if and only if $\gamma \geq \eta + \varepsilon$;
- no other pair of outcomes of distinct alternatives is ranked.

Reflexivity and Monotonicity in Money follow from the first clause, since outcomes of the same alternative are ranked by money only. *Transitivity.* A chain through an alternative other than x and y stays at that alternative and reduces to the first clause. Composing the first clause with the second on either side preserves $\eta \geq \gamma + \varepsilon$, because it adds money to

the x-outcome or removes money from the y-outcome. Composing two pairs of the second clause, $(x,\eta) \succsim_\varepsilon^+ (y,\gamma) \succsim_\varepsilon^+ (x,\eta')$, gives $\eta \geq \eta' + 2\varepsilon$, so $(x,\eta) \succsim_\varepsilon^+ (x,\eta')$; the chain through an x-outcome is symmetric. *Thick incomparability.* $(x,\eta) \bowtie_\varepsilon^+ (y,\gamma)$ if and only if $|\eta - \gamma| < \varepsilon$, so both sets in Assumption 7 are open intervals; for outcomes of any other two distinct alternatives both sets equal $\mathbb{R}$. Let $\succsim$ on X be the restriction of $\succsim_\varepsilon^+$ to zero money, as in Assumption 5. Hence $\succsim_\varepsilon^+ \in \mathcal{R}$, and each of the four thicknesses $d_{\pm x}^{\succsim_\varepsilon^+}(x,y)$ and $d_{\pm y}^{\succsim_\varepsilon^+}(x,y)$ equals ε, so $d^{\succsim_\varepsilon^+}((x,0),(y,0)) = \varepsilon$.

Step 1: all band preferences with $\varepsilon < \Delta$ share one truthful report. Fix $\varepsilon \in (0,\Delta)$ and a task $\{d_1, d_2\} \in D$. If d_1 and d_2 are outcomes of the same alternative, they differ in money and are strictly ranked by it. If $d_1 = (x,\eta)$ and $d_2 = (y,\gamma)$, they are incomparable when $\eta = \gamma$, and strictly ranked by money when $\eta \neq \gamma$, since then $|\eta - \gamma| \geq \Delta > \varepsilon$. Otherwise they are incomparable. None of these cases depends on ε, and $\succsim_\varepsilon^+$ has no indifference between distinct outcomes. Hence $T(\succsim_\varepsilon^+) = \{m_0\}$ for a single report m_0 that is the same for every $\varepsilon \in (0,\Delta)$. In particular, $m_0 \in M_R$ and m_0 reports $(x,0) \hat{\bowtie} (y,0)$.

Step 2: a fixed positive margin. By Proposition 1, $(x,0)$ and $(y,0)$ coexist under (ϕ, m_0): there are states ω_x, ω_y with

$$\phi(m_0)(\omega_x) = (x,\eta^-), \quad (y,\gamma^+) \in A(\phi,\omega_x), \qquad \phi(m_0)(\omega_y) = (y,\gamma^-), \quad (x,\eta^+) \in A(\phi,\omega_y),$$

where $\eta^- \leq 0 \leq \eta^+$, $\gamma^- \leq 0 \leq \gamma^+$, and at least one inequality is strict. Hence

$$\kappa = \max\{\gamma^+ - \eta^-,\ \eta^+ - \gamma^-\} > 0.$$

The states and outcomes depend only on ϕ and m_0, so κ does not depend on ε.

Step 3: contradiction. Choose $\varepsilon \in (0, \min\{\Delta, \kappa\})$. If $\kappa = \eta^+ - \gamma^-$, then $\eta^+ > \gamma^- + \varepsilon$, so $(x,\eta^+) \succ_\varepsilon^+ (y,\gamma^-)$: in ω_y an outcome within reach is strictly better under $\succsim_\varepsilon^+$ than the outcome m_0 pays. If $\kappa = \gamma^+ - \eta^-$, the same holds in ω_x with $(y,\gamma^+) \succ_\varepsilon^+ (x,\eta^-)$. Either way $m_0 \notin \bar{M}(\phi, \succsim_\varepsilon^+)$, although $m_0 \in T(\succsim_\varepsilon^+)$ by Step 1. This violates (a) of Lemma 2 at $\succsim_\varepsilon^+$, a contradiction. Step 3 is the argument behind Corollary 1, applied to a DM whose thickness ε is smaller than the margin. □

Proof of Lemma 3. The proof follows that of Lemma 2, with incomparable-side lies in place of comparable-side lies. A report contains an incomparable-side lie exactly when it declares a ranking on some core task $\{(x,0),(y,0)\}$ with $(x,0) \bowtie^+ (y,0)$. Hence (b) says: $\forall m^\star \in T(\succsim^+)$ and $\forall m'$ that contains an incomparable-side lie, $\exists \omega \in \Omega$ with $\phi(m^\star)(\omega) \succ^+ \phi(m')(\omega)$.

By Definition 5, ϕ satisfies ISIC if and only if, $\forall \succsim^+ \in \mathcal{R}$, conditions 1 and 2 hold $\forall m^\star \in T(\succsim^+)$. Fix such a $\succsim^+$ and write $\mathcal{C} = \mathcal{C}(\succsim^+)$. It suffices to show that, for this $\succsim^+$, conditions 1 and 2 hold $\forall m^\star \in T(\succsim^+)$ if and only if (a) and (b) hold.

Condition 1 and (a). For a given $m^\star \in T(\succsim^+)$, condition 1 states that $\phi(m) \not\succ^\star \phi(m^\star)$ for every $m \in M$ and every $\succsim^\star \in \mathcal{C}$. By the first statement of Lemma 1, applied to $m = m^\star$, this holds if and only if $m^\star \in \bar{M}(\phi, \succsim^+)$. Hence condition 1 holds $\forall m^\star \in T(\succsim^+)$ if and only if $T(\succsim^+) \subseteq \bar{M}(\phi, \succsim^+)$, which is (a).

Condition 2 implies (b). Fix $m^\star \in T(\succsim^+)$ and m' that contains an incomparable-side lie. By condition 2, some $\succsim^\star \in \mathcal{C}$ can compare $\phi(m^\star)$ and $\phi(m')$, and it strictly prefers $\phi(m^\star)$. Assumption 11(ii) then gives a state ω with $\phi(m^\star)(\omega) \succ^+ \phi(m')(\omega)$. Since $m^\star$ and m' were arbitrary, (b) holds.

(a) and (b) imply condition 2. Fix $m^\star \in T(\succsim^+)$ and m' that contains an incomparable-side lie. By (b), some ω_0 has $\phi(m^\star)(\omega_0) \succ^+ \phi(m')(\omega_0)$.

- *Some extension can compare the two acts.* Apply Lemma A.1 with $f = \phi(m^\star)$ and $G = \{\phi(m')\}$: its condition (a) holds with $\omega_g = \omega_0$, so $\exists \succsim^\star \in \mathcal{C}$ with $\phi(m^\star) \succ^\star \phi(m')$.
- $\forall \succsim^\star \in \mathcal{C}$ *that can compare them,* $\phi(m^\star) \succ^\star \phi(m')$. By (a), $m^\star \in \bar{M}(\phi, \succsim^+)$. Together with ω_0, Lemma A.2 gives $\phi(m') \not\succsim^\star \phi(m^\star)$ $\forall \succsim^\star \in \mathcal{C}$. If $\succsim^\star$ can compare the two acts, it must therefore have $\phi(m^\star) \succsim^\star \phi(m')$ and not $\phi(m') \succsim^\star \phi(m^\star)$, that is, $\phi(m^\star) \succ^\star \phi(m')$.

Hence condition 2 holds $\forall m^\star \in T(\succsim^+)$. Combining the three parts $\forall \succsim^+ \in \mathcal{R}$ gives the lemma. □

Proof of Proposition 3. Suppose ϕ satisfies ISIC and fix a core task $\{(x, 0), (y, 0)\}$. Let $\succsim_1^+ = R_0$, the relation that compares outcomes of the same alternative by money and leaves outcomes of different alternatives incomparable. It belongs to $\mathcal{R}$ (both sets in Assumption 7 equal $\mathbb{R}$) and $(x, 0) \bowtie_1^+ (y, 0)$. Let $\succsim_2^+$ rank outcomes by money alone: $(a, \alpha) \succsim_2^+ (b, \beta)$ if and only if $\alpha \geq \beta$. It is complete, transitive and strictly increasing in money, so it belongs to $\mathcal{R}$, and $\succ_1^+ \subseteq \succ_2^+$.

Let $m_1 \in T(\succsim_1^+)$ and $m_2 \in T(\succsim_2^+)$. Since $(x, 0) \sim_2^+ (y, 0)$, m_2 reports a ranking on this core task, so m_2 contains an incomparable-side lie under $\succsim_1^+$. By (b) of Lemma 3 at $\succsim_1^+$, some ω has $\phi(m_1)(\omega) \succ_1^+ \phi(m_2)(\omega)$, hence $\phi(m_1)(\omega) \succ_2^+ \phi(m_2)(\omega)$. So $m_2 \notin \bar{M}(\phi, \succsim_2^+)$, which contradicts (a) of Lemma 3 at $\succsim_2^+$. □

Proof of Proposition 4. Suppose ϕ satisfies ISICWT, and let $\succsim_1^+$ and $\succsim_2^+$ be as in the proof of Proposition 3; $\succsim_1^+$ is incomplete in D_{core} and $\succsim_2^+$ is complete. Condition 3 at $\succsim_1^+$ gives

$m \in M_{ic}$ such that, by Lemma A.1, $\forall m' \in M_c$, $\exists \omega_{m'} \in \Omega$ with $\phi(m)(\omega_{m'}) \succ_1^+ \phi(m')(\omega_{m'})$, hence $\phi(m)(\omega_{m'}) \succ_2^+ \phi(m')(\omega_{m'})$. Applying Lemma A.1 under $\succsim_2^+$ with $f = \phi(m)$ and $G = \{\phi(m') : m' \in M_c\}$ gives a single $\succsim^\star \in \mathcal{C}(\succsim_2^+)$ with $\phi(m) \succ^\star \phi(m')\ \forall m' \in M_c$. For this $\succsim^\star$, no $m' \in M_c$ satisfies $\phi(m'') \not\succ^\star \phi(m')\ \forall m'' \in M_{ic}$ (take $m'' = m$), which contradicts condition 1 at $\succsim_2^+$. □

A.3 An Example of a *b-RCSIC* Mechanism using Random Incentive System – Status Quo Updating Mechanism

Experiment Structure Fix the target core comparisons and choose $b > 0$. Let $X_0 \subseteq X$ be the set of alternatives that appear in some target pair. For each $x \in X_0$, choose one auxiliary outcome (x, η_x) with $0 < |\eta_x| < b$. Let

$$O^+ = \{(x, 0) : x \in X_0\} \cup \{(x, \eta_x) : x \in X_0\}.$$

Define D to contain every unordered pair of distinct outcomes in O^+. Payment lists may be any partial permutation of O^+.

Mechanism Design let C be any subset of O^+, $C \subseteq O^+$. A *partial permutation* is a bijection $\sigma : \{1, \ldots, |C|\} \to C$. After participants answer all the tasks in D, the mechanism randomly select a C and a partial permutation of it. The mechanism processes σ sequentially:

1. Initialize the current option $c := \sigma(1)$.
2. For $t = 2, \ldots, |C|$: compare c with $\sigma(t)$ under the reported relation $\hat{\succsim}$; if $\sigma(t) \hat{\succsim} c$, set $c := \sigma(t)$; otherwise keep c.
3. The final payoff is the current option c after $\sigma(|C|)$ is processed.
4. If $(x, 0) \hat{\bowtie} (y, 0)$ is declared by a participant, then the mechanism automatically views that $(x, \eta') \hat{\bowtie} (y, 0)$ and $(x, 0) \hat{\bowtie} (y, \gamma')$, where $|\eta'| < b$ and $|\gamma'| < b$ are also declared. I call this rule the *binding rule*. For a report m, write $\hat{m}$ for m together with the declarations that the binding rule adds.

Let Ω be the set of all such σ, and let $\phi(m)(\sigma)$ be the final payoff on σ under the report m.

Remark 1. *The Status Quo Updating Mechanism evaluates each list of options according to the reported preference. The preferred one is always chosen, and the status quo option is retained when the alternative is incomparable with the status quo. By transitivity, the final option is always a maximal element given any list (also see Mu (2021) lemma 1 for this*

result [11]*). This mechanism design is related to the status quo choice procedure in Mu (2021). Mu (2021) takes the status quo choice procedure as given and studies the rationalizability of choice functions by incomplete preference using the revealed preference method, which is from choice to preference. This mechanism targets the incentive compatibility and uses the status quo procedure to select the maximum results for subjects, which is from preference to choice. In addition, the binding rule in (4) bears no cost on DMs who genuinely can not compare while punishing DMs who can compare but report incomplete preferences*

Proposition 8. *The Status Quo Updating mechanism satisfies b-RCSIC (Definition 4).*

Proof of Proposition 8. *Step 1 (proof of (a)).* Fix $\succsim^+ \in \mathcal{P}_b$, $m^\star \in T(\succsim^+)$ and $\sigma \in \Omega$. Every declaration of $\hat{m}^\star$ is truthful. Those of $m^\star$ are truthful by definition. The binding rule adds $(x,\eta')\hat{\bowtie}(y,0)$ and $(x,0)\hat{\bowtie}(y,\gamma')$ only when $m^\star$ declares $(x,0)\hat{\bowtie}(y,0)$, that is, when $(x,0) \bowtie^+ (y,0)$; since $\succsim^+ \in \mathcal{P}_b$, the thickness of this pair is at least b in each direction, $(x,\eta') \bowtie^+ (y,0)$ and $(x,0) \bowtie^+ (y,\gamma')$ for all $|\eta'| < b$ and $|\gamma'| < b$.

Let c_t be the current option after $\sigma(t)$ is processed and $S_t = \{\sigma(1), \ldots, \sigma(t)\}$. We show by induction that $c_t \in \text{Maximal}(S_t, \succsim^+)$. For $t = 1$ this is immediate. Suppose it holds for $t-1$ and let $p = \sigma(t)$.

- If $\hat{m}^\star$ declares $p \hat{\succsim} c_{t-1}$, then $c_t = p$, and truthfulness gives $p \succsim^+ c_{t-1}$. If some $q \in S_t$ had $q \succ^+ p$, then $q \in S_{t-1}$, and $q \succ^+ p \succsim^+ c_{t-1}$ would give $q \succ^+ c_{t-1}$ by transitivity, contradicting the induction hypothesis.
- Otherwise $c_t = c_{t-1}$. A truthful report declares $p \hat{\succsim} c_{t-1}$ whenever $p \succ^+ c_{t-1}$, so $p \not\succ^+ c_{t-1}$; and by the induction hypothesis no element of S_{t-1} is strictly better than c_{t-1}.

Hence $\phi(m^\star)(\sigma) \in \text{Maximal}(C, \succsim^+)$, where C is the set of options listed by σ. Every payment on σ is an entry of σ, so $A(\phi,\sigma) \subseteq C$, and therefore $\phi(m')(\sigma) \not\succ^+ \phi(m^\star)(\sigma)$ for all $m' \in M$ and $\sigma \in \Omega$; that is, $m^\star \in \bar{M}(\phi, \succsim^+)$.

Step 2 (proof of (b)). Fix $\succsim^+ \in \mathcal{P}_b$, a report m' that contains a comparable-side lie, and $m^\star \in T(\succsim^+)$. Some core task $D_i = \{(x,0),(y,0)\}$ whose options are comparable under $\succsim^+$ has $m_i' \notin T_i(\succsim^+)$. Up to relabelling x and y there are two cases.

Case 1: $(x,0) \succ^+ (y,0)$ and $m_i' \in \{(y,0)\hat{\succsim}(x,0),\ (x,0)\hat{\bowtie}(y,0)\}$. Take $\sigma = ((y,0),(x,0))$. Since m' does not declare $(x,0)\hat{\succsim}(y,0)$, the status quo is kept and $\phi(m')(\sigma) = (y,0)$. Since $(x,0) \succ^+ (y,0)$, $m^\star$ declares $(x,0)\hat{\succsim}(y,0)$, so $\phi(m^\star)(\sigma) = (x,0)$, and $\phi(m^\star)(\sigma) \succ^+ \phi(m')(\sigma)$.

[11] Mu (2021) focuses on strict partial order while we allow weak partial order.

Case 2: $(x,0) \sim^+ (y,0)$ and $m'_i = (x,0)\hat{\bowtie}(y,0)$. The design gives $(x,\eta) \in O^+$ with $\eta = \eta_x$ and $0 < |\eta| < b$, so $\{(x,\eta),(y,0)\} \in D$. Because $|\eta| < b$, the binding rule makes $\hat{m}'$ declare $(x,\eta)\hat{\bowtie}(y,0)$. The binding rule does not change $\hat{m}^\star$ on this task: the only declaration that binds it is $(x,0)\hat{\bowtie}(y,0)$, which $m^\star$ does not make because $(x,0) \sim^+ (y,0)$.

- If $\eta > 0$, take $\sigma = ((y,0),(x,\eta))$. The status quo is kept under $\hat{m}'$, so $\phi(m')(\sigma) = (y,0)$. Since $(x,\eta) \succ^+ (x,0) \sim^+ (y,0)$, monotonicity and transitivity give $(x,\eta) \succ^+ (y,0)$, so $\hat{m}^\star$ declares $(x,\eta)\hat{\succsim}(y,0)$ and $\phi(m^\star)(\sigma) = (x,\eta)$.
- If $\eta < 0$, take $\sigma = ((x,\eta),(y,0))$. The status quo is kept under $\hat{m}'$, so $\phi(m')(\sigma) = (x,\eta)$. Since $(y,0) \sim^+ (x,0) \succ^+ (x,\eta)$, transitivity gives $(y,0) \succ^+ (x,\eta)$, so $\hat{m}^\star$ declares $(y,0)\hat{\succsim}(x,\eta)$ and $\phi(m^\star)(\sigma) = (y,0)$.

In every case $\phi(m^\star)(\sigma) \succ^+ \phi(m')(\sigma)$, which is (b).

By (a), (b) and Lemma A.3, the Status Quo Updating mechanism is b-RCSIC. □

A.4 Proof of Results in Section 5

Proof for Proposition 6:

Proof by contradiction. If Proposition 6 does not hold, $\exists \succsim^+ \in \mathcal{P}_b$, a list L, and a truthful report sequence $m \in T(\succsim^+)$ on L, s.t. $\exists l_j \in \text{Maximal}(\{l_0, l_1, \ldots, l_i\}, \succsim^+)$ s.t. $l_j \succ^+ c_i$, where c_i is the current option in round i under m. If the list end here, $\phi(m) = c_i$. Given that l_j is one of the option in some round, it can be attained by some message m'. Therefore, $\phi(m') \succ^+ \phi(m)$ and violates condition 1 in Definition 7.

Proof of Proposition 5. Fix $b > 0$ and a single-list extended ABC mechanism ϕ that satisfies condition 1 of Definition 7. We show that condition 2 fails.

Truthful payments are maximal. For a list L, let $E(L)$ be its set of elements. Any $l_j \in E(L)$ can be made the payment: the report sequence that declares l_j preferred in the opening comparison D_i^1 of every round $i \geq j$ ends each of these rounds with l_j as the current choice, since a ranking ends the round. Condition 1 therefore gives $\phi(m^\star) \in \text{Maximal}(E(L), \succsim^+)$ for every $\succsim^+ \in \mathcal{P}_b$ and every $m^\star \in T(\succsim^+)$ on L.

Prove by construction.

step 1: *construct two preferences.* For distinct $u, v, z \in X$ and $w \in \{u, v\}$, let $R_w = \{((w,\eta),(z,\zeta)) : \eta \geq 0 \geq \zeta\}$, and let $\succsim_1^+ = R_0 \cup R_u \cup R_v$ and $\succsim_2^+ = R_0 \cup R_v$, with R_0 as in the proof of Proposition 1. Each R_w has the form of R_{xy} there, and no two pairs of

$R_u \cup R_v$ compose, since each ends at a z-outcome and none starts at one. Step 1 of that proof therefore shows that both relations belong to $\mathcal{R}$, and that two zero-money outcomes that are incomparable stay incomparable at every money level, so both relations belong to $\mathcal{P}_b$. Moreover, for every $S \subseteq X \times \mathbb{R}$ with $(v, 0) \in S$,

$$\text{Maximal}(S, \succsim_1^+) = \text{Maximal}(S, \succsim_2^+). \tag{$\dagger$}$$

Indeed, $\succ_2^+ \subseteq \succ_1^+$, and the only strict pairs of $\succsim_1^+$ missing from $\succsim_2^+$ are those in R_u, which beat some (z, ζ) with $\zeta \leq 0$; every such (z, ζ) is also beaten by $(v, 0) \in S$ under $\succsim_2^+$.

step 2: *Construct a lie.* Pick distinct $x, y, z \in X$ and a list L beginning with $(x, 0), (y, 0), (z, 0)$. Round 1 compares only $(x, 0)$ and $(y, 0)$, which are incomparable under $\succsim_1^+$ and under $\succsim_2^+$ for either assignment of x, y to u, v. A truthful DM therefore declares $\hat{\bowtie}$ in every task of round 1, and the round ends with $c_1 = (u, 0)$ for some $u \in \{x, y\}$. Let v be the other element of $\{x, y\}$, and define $\succsim_1^+$ and $\succsim_2^+$ with these u, v, z ($\succsim_1^+$ does not depend on which of x, y is u). Round 2 presents $\{(u, 0), (z, 0)\} \in D_{core}$. Since $(u, 0) \succ_1^+ (z, 0)$, the declaration $(u, 0) \hat{\bowtie} (z, 0)$ is a comparable-side lie under $\succsim_1^+$, while it is truthful under $\succsim_2^+$. Hence the report sequence m' that is $\succsim_2^+$-truthful throughout contains a comparable-side lie under $\succsim_1^+$.

step 3: *Show that no continuation penalizes the lie.* Let L' be any continuation of L after round 2, let $m^\star \in T(\succsim_1^+)$ be a truthful report sequence on L' (it exists, since every task admits a truthful declaration), and let m'' be a report sequence on L' that agrees with m' up to and including the lie in round 2 and is $\succsim_2^+$-truthful throughout. By the first paragraph, $\phi(m^\star)$ is $\succsim_1^+$-maximal and $\phi(m'')$ is $\succsim_2^+$-maximal in $E(L')$. Since $(v, 0) \in E(L')$, ($\dagger$) makes both payments $\succsim_1^+$-maximal, so $\phi(m^\star) \not\succ_1^+ \phi(m'')$. Condition 2 thus fails at $\succsim_1^+ \in \mathcal{P}_b$. □

Within a list, write $S_i = \{l_0, \ldots, l_i\}$ for the options revealed up to round i.

Lemma A.5. *Let $\succsim^+ \in \mathcal{R}$, and run AMC-1 on a list L with declarations that are all truthful for $\succsim^+$. Then $c_i \in \text{Maximal}(S_i, \succsim^+)$ for every round i. In particular, AMC-1 is an AMC mechanism.*

Proof. By induction on i. For $i = 0$, $c_0 = l_0$ is the only element of S_0. Suppose $c_{i-1} \in \text{Maximal}(S_{i-1}, \succsim^+)$, so that no element of S_{i-1} is strictly better than c_{i-1}.

Case 1: $l_i \hat{\succsim} c_{i-1}$ is declared, so $c_i = l_i$. Given truthful report, $l_i \succsim^+ c_{i-1}$. Suppose some $s \in S_i$ had $s \succ^+ l_i$. Then $s \neq l_i$, so $s \in S_{i-1}$, and $s \succ^+ l_i \succsim^+ c_{i-1}$ gives $s \succ^+ c_{i-1}$ by transitivity, contradicting the induction hypothesis.

Case 2: otherwise, so $c_i = c_{i-1}$. Since $l_i \hat{\succsim} c_{i-1}$ is not declared, truthful report gives $l_i \not\succ^+$

c_{i-1}. By the induction hypothesis no element of S_{i-1} is strictly better than c_{i-1}. Hence $c_i \in \text{Maximal}(S_i, \succsim^+)$. Every truthful report sequence $m \in T(\succsim^+)$ on L makes only truthful declarations, so the first statement applies. □

Lemma A.6. *Let $\succsim^+ \in \mathcal{R}$, and run AMC-2 on a list L with declarations that are all truthful for $\succsim^+$. Then, for every round i:*

(I1) $c_i \in \text{Maximal}(S_i, \succsim^+)$;

(I2) $\forall s \in S_i \setminus (W_i \cup \{c_i\})$, $\exists k \in W_i \cup \{c_i\}$ *with* $k \succsim^+ s$.

In particular, AMC-2 is an AMC mechanism.

(I2) says that every element the mechanism has set aside is weakly below the current choice or below an element kept in the buffer. This is what allows the buffer rule to switch to l_i after an incomparability report: any element that could be strictly better than l_i is checked against l_i before the switch.

Proof. By induction on i. For $i = 0$, $c_0 = l_0$, $W_0 = \emptyset$ and $S_0 \setminus (W_0 \cup \{c_0\}) = \emptyset$, so (I1) and (I2) hold. Suppose they hold at $i - 1$.

(case 1): $c_{i-1} \hat{\succsim} l_i$, *so* $c_i = c_{i-1}$ *and* $W_i = W_{i-1}$. (I1): by truthful report, $l_i \not\succ^+ c_{i-1}$, and no element of S_{i-1} is strictly better than c_{i-1}. (I2): the only new element of $S_i \setminus (W_i \cup \{c_i\})$ is l_i, and $c_i \succsim^+ l_i$ given truthful report.

(case 2): $l_i \hat{\succsim} c_{i-1}$, *so* $c_i = l_i$ *and* $W_i = W_{i-1}$. (I1): given truthful report, $l_i \succsim^+ c_{i-1}$. If some $s \in S_i$ had $s \succ^+ l_i$, then $s \in S_{i-1}$ and $s \succ^+ c_{i-1}$ by transitivity, contradicting (I1) at $i - 1$. (I2): the element c_{i-1} leaves $W \cup \{c\}$, and $l_i \succsim^+ c_{i-1}$. An element s that was covered by c_{i-1}, that is $c_{i-1} \succsim^+ s$, is now covered by l_i, since $l_i \succsim^+ c_{i-1} \succsim^+ s$. Elements covered by $W_{i-1} = W_i$ stay covered.

(case 3 (i), keep): $c_{i-1} \hat{\bowtie} l_i$ *and some* $w \hat{\succsim} l_i$ *with* $w \in W_{i-1}$, *so* $c_i = c_{i-1}$ *and* $W_i = W_{i-1}$. (I1): given truthful report, $c_{i-1} \bowtie^+ l_i$, so $l_i \not\succ^+ c_{i-1}$, and no element of S_{i-1} is strictly better than c_{i-1}. (I2): the new element l_i is covered by $w \in W_i$, since $w \succsim^+ l_i$ by truthful report.

(case 3 (ii), switch): $c_{i-1} \hat{\bowtie} l_i$ *and no* $w \hat{\succsim} l_i$ *with* $w \in W_{i-1}$, *so* $c_i = l_i$ *and* $W_i = W_{i-1} \cup \{c_{i-1}\}$. (I2): c_{i-1} moves into W_i, so everything it covered stays covered, and elements covered by $W_{i-1} \subseteq W_i$ stay covered. (I1): suppose some $s \in S_i$ had $s \succ^+ l_i$; then $s \in S_{i-1}$.

- $s = c_{i-1}$ is impossible: by truthful report, $c_{i-1} \bowtie^+ l_i$.

- $s \in W_{i-1}$ is impossible: the pair $\{s, l_i\}$ was presented, and by truthful report, the declaration was $s \hat{\succsim} l_i$, so the round would have ended with a keep.
- Otherwise $s \in S_{i-1} \setminus (W_{i-1} \cup \{c_{i-1}\})$. By (I2) at $i-1$, some $k \in W_{i-1} \cup \{c_{i-1}\}$ has $k \succsim^+ s$, so $k \succ^+ l_i$ by transitivity. If $k = c_{i-1}$, this contradicts $c_{i-1} \bowtie^+ l_i$; if $k \in W_{i-1}$, the previous bullet applies to k.

Hence $l_i \in \text{Maximal}(S_i, \succsim^+)$. Every truthful report sequence $m \in T(\succsim^+)$ on L makes only truthful declarations, so the first statement applies. □

Remark 2. *The proof uses the buffer only through (I2). It therefore also covers variants that keep more elements in W (for instance, adding l_i to W after a keep in item 3 of AMC-2), or that remove an element w from W once some element of $W \cup \{c\}$ has been declared weakly above it. Both changes preserve (I2).*

Proof for Proposition 7. We show that both *Multi-list AMC-1* and *Multi-list AMC-2* are b-RCSIC. Fix $\succsim^+ \in \mathcal{P}_b$.

Condition 1. The payment is the final current choice of the last list L^K, which, by the reset rule, is processed afresh by AMC-1 or AMC-2. Along a truthful report sequence, every declaration in force is truthful: those made by the DM by definition, and those imposed by the binding rule because, for $\succsim^+ \in \mathcal{P}_b$, $(x, 0) \bowtie^+ (y, 0)$ implies $(x, \eta') \bowtie^+ (y, 0)$ and $(x, 0) \bowtie^+ (y, \gamma')$ $\forall\, |\eta'| < b$ and $\forall\, |\gamma'| < b$. Since AMC-1 and AMC-2 are AMC rules, the truthful payment is maximal in L^K. Every report's payment is an element of L^K, so no report pays strictly more than truth-telling.

Condition 2. Suppose the DM lies on the comparable side in a core task $\{p, q\}$ in round i of list L^k, in the opening comparison D_i^1 or, under AMC-2, in a comparison with the buffer. Consider the continuation that agrees with the lists presented so far up to round i of L^k, ends L^k there, and adds one final list L^K; it is a valid continuation because the DM knows neither the number nor the length of the lists. By the binding rule, the lie is in force in L^K whatever the DM reports after it. Let m'' be any continuation of the lie and $m^\star$ any truthful report sequence.

Case 1: $p \succ^+ q$ but the DM reports $q \hat{\succsim} p$. Let L^K start with (p, q), followed by elements that are strictly worse than q (possibly none). By binding, $c_1^K = q$. Since p does not reappear in L^K, the current choice afterwards is q or a later element, so $q \succsim^+ \phi(m'')$. Truth-telling declares $p \hat{\succsim} q$ and keeps p, which is strictly better than every later element, so $\phi(m^\star) = p$. Hence $\phi(m^\star) = p \succ^+ q \succsim^+ \phi(m'')$, and $\phi(m^\star) \succ^+ \phi(m'')$.

Case 2: $q \succ^{+} p$ but the DM reports $p \hat{\succsim} q$. This is Case 1 with the roles of p and q exchanged.

Case 3: $q \succsim^{+} p$ but the DM reports $p \hat{\bowtie} q$. Write $q = (x, 0)$ and $p = (y, 0)$, and pick $\gamma \in (-b, 0)$ such that (y, γ) does not appear in any list before the lie. By binding, $(x, 0) \hat{\bowtie} (y, \gamma)$ is in force for the liar. Since $(x, 0) \succsim^{+} (y, 0) \succ^{+} (y, \gamma)$, we have $(x, 0) \succ^{+} (y, \gamma)$, and a truthful DM declares $(x, 0) \hat{\succsim} (y, \gamma)$. (i) Multi-list AMC-1: let $L^K = ((y, \gamma), (x, 0))$. After $\hat{\bowtie}$ the status quo is kept, so $\phi(m'') = (y, \gamma)$, while truth-telling switches to $(x, 0)$. (ii) Multi-list AMC-2: let $L^K = ((x, 0), (y, \gamma))$. After $\hat{\bowtie}$ the buffer is empty (reset), so the mechanism switches to (y, γ) and $\phi(m'') = (y, \gamma)$, while truth-telling keeps $(x, 0)$. In both, $\phi(m^\star) = (x, 0) \succ^{+} (y, \gamma) = \phi(m'')$.

Case 4: $p \succsim^{+} q$ but the DM reports $p \hat{\bowtie} q$. This is Case 3 with the roles of p and q exchanged.

Hence some multi-list AMC mechanisms attain b-RCSIC.

□

B Experiment Interface

This appendix shows the screens subjects see during the experiment. I show screens of *Multi-list AMC-1* as the benchmark, in the order in which subjects meet them. For *Multi-list AMC-2* and *Multi-list ABC*, I show only the instructions and the comprehension quiz.

B.1 Multi-list AMC-1 Mechanism

Instructions (1 of 3).

Instructions

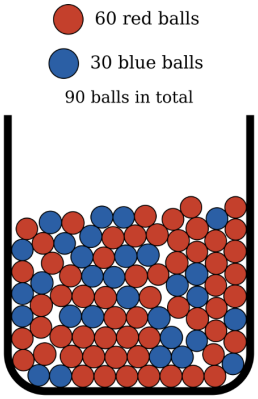
Please read the instructions carefully. The Next button will become available after 3 minutes.

Introduction

In this experiment you will make some choices between two options. At the end, one option will be played for real money.

What an option is

Each option is a **bet on the colour of a ball drawn from an urn**, together with a **sure payment** you receive no matter which ball is drawn. The urn's contents are always shown on your screen. For example:



Urn 1

Option X

A ball is drawn from Urn 1.

red	$5.00
blue	$0.00
Sure payment, whatever the ball	+ $0.50

If the ball drawn from Urn 1 is red you receive $5.00 from the bet, otherwise the bet pays nothing, and either way you also receive $0.50 for sure.

Instructions (2 of 3).

Making choices

Your choices are organized into **sequences** of rounds. Within each sequence, options are numbered in the order in which they appear: Option 1, Option 2, Option 3, and so on.

At the beginning of each sequence, you start by holding **Option 1** and compare it with **Option 2**. In each later round, the option you currently hold is compared with a new option.

For example, in a round you see Option X and Option Y side by side, and you are asked to choose one of:

- "I rank Option X above Option Y"
- "I rank Option Y above Option X"
- "I rank the two options exactly the same"
- "I do not know how I rank Option X and Option Y"

There are no right or wrong answers. Choose the answer that best describes your preference.

How your choice determines what you hold

On each screen, the option you currently hold is shown on the left, and a new option is shown on the right.

For example, suppose you currently hold Option X, and the new option is Option Y:

- If you choose "I rank X above Y", you continue to hold X.
- If you choose "I rank Y above X", you replace X with Y.
- If you choose "I rank the two options exactly the same", you replace X with Y.
- If you choose "I do not know how I rank X and Y", you continue to hold X.

In other words, your current option is replaced only when you rank the new option above it or exactly the same as it. Otherwise, your current option stays.

The option you hold becomes your current option in the next round. It is compared with the next new option, and the same process continues until the sequence ends.

Sequence structure

The experiment is divided into sequences. When a new sequence begins, the option you were holding is reset. You start the new sequence by holding **Option 1** and comparing it with **Option 2**. You continue this process until the end.

What you know and do not know

At the start of the experiment, you are not told how many sequences there are or how many rounds each sequence contains. Therefore, the experiment may end after any round.

In addition, you will not know which options are still to come. You will only see the options that you have already encountered and the two options shown on your current screen. The set of options is determined in advance by the experimenter, and you will encounter all of the options in the experiment.

Repeated comparisons

An option can appear more than once, so a comparison you answer early may appear again at any later point in the experiment. If you are again asked how you rank two options, and you have already answered for that pair, you will see your earlier answer already recorded and you cannot change it, even if the options appear in a different order. Therefore, answer each comparison according to your true preference.

Instructions (3 of 3).

Comparisons with small payment changes

If you choose "I do not know how I rank the two options" for a pair of bets, your answer will also apply to a later comparison involving the same two bets if the sure payment attached to one of the options has changed by less than $0.25.

For example, suppose you answer "I do not know how I rank the two options" for:

- Option A: a bet on red from urn 1, prize $6, plus $0.00 for sure
- Option B: a bet on blue from urn 1, prize $6, plus $0.00 for sure

Later, you may see the same two bets, but Option A now offers $0.10 for sure:

- Option A: a bet on red from urn 1, prize $6, plus **$0.10** for sure
- Option B: a bet on blue from urn 1, prize $6, plus $0.00 for sure

Because the sure payment changed by $0.10, which is less than $0.25, if you are asked to rank the two options, your previous "I do not know how I rank the two options" answer will already be recorded and you cannot change the answer.

This rule applies only when your original answer was "I do not know how I rank Option X and Option Y". **In other words, by saying you do not know how you rank two options, you are also saying that adding or removing a sure payment of less than $0.25 to one of them would not settle the matter for you. Answering truthfully that you do not know how you rank two options will not hurt you.**

However, if you do know how you rank two options and still choose that answer, it may hurt you. For example, suppose the two options in a round are:

- Option A: a bet on red from an urn with 50 red and 50 blue balls, prize $6, plus $0.00 for sure
- Option B: a bet on blue from the same urn, prize $6, plus $0.00 for sure

Most people rank these two options exactly the same. Suppose you nevertheless answer "I do not know how I rank Option A and Option B". Later, you may see the same two options again, but Option A now offers $0.10 for sure:

- Option A: a bet on red from the same urn, prize $6, plus **$0.10** for sure
- Option B: a bet on blue from the same urn, prize $6, plus $0.00 for sure

If you are asked to rank the two options, because of your earlier answer this comparison is already answered for you and you cannot change it, even though Option A is now clearly better than Option B.

Payment

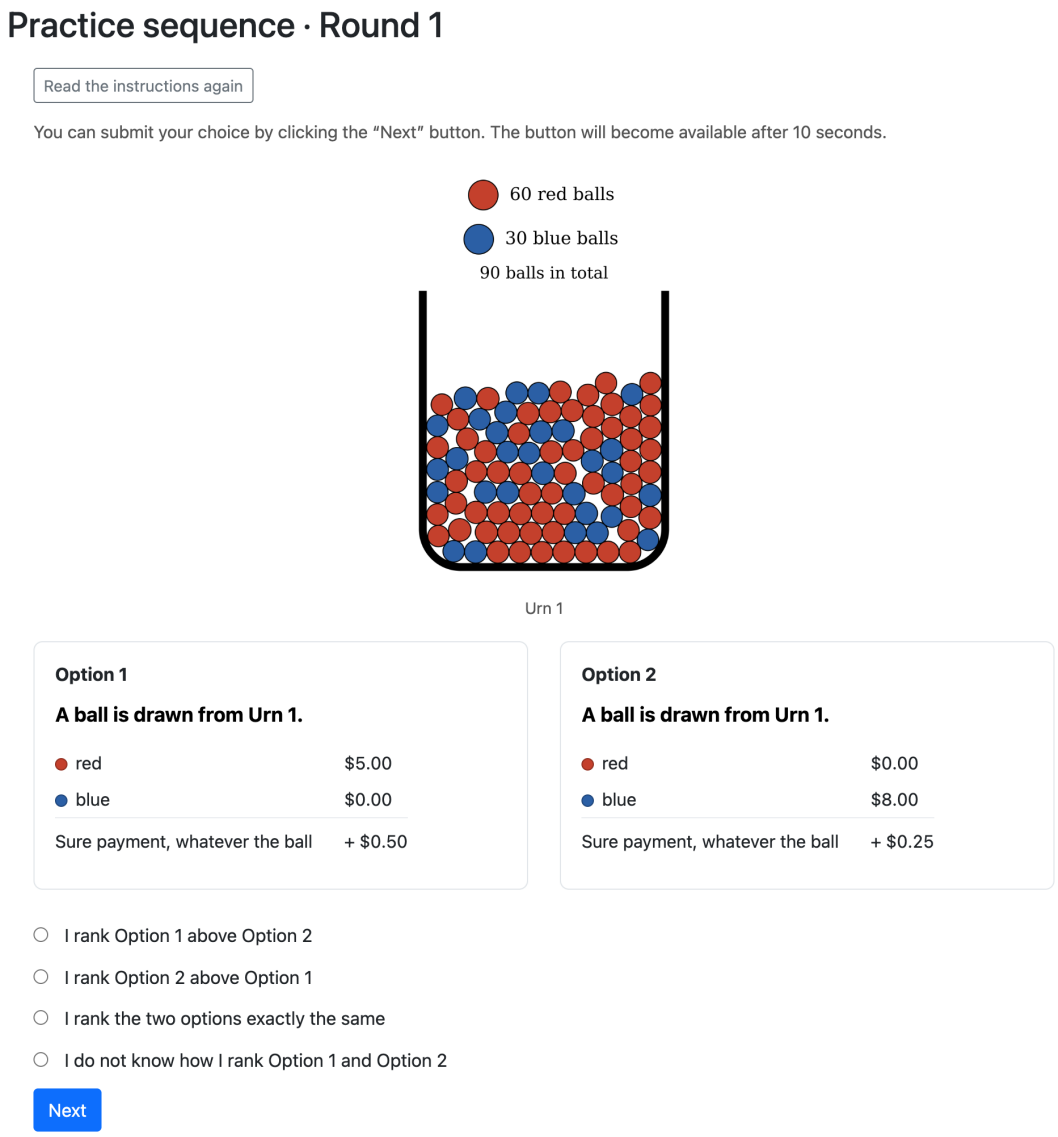
Whichever option you are holding when the experiment ends is your payoff. A ball is then drawn from that option's urn, and you are paid the prize for the colour drawn plus that option's sure payment.

Your best strategy

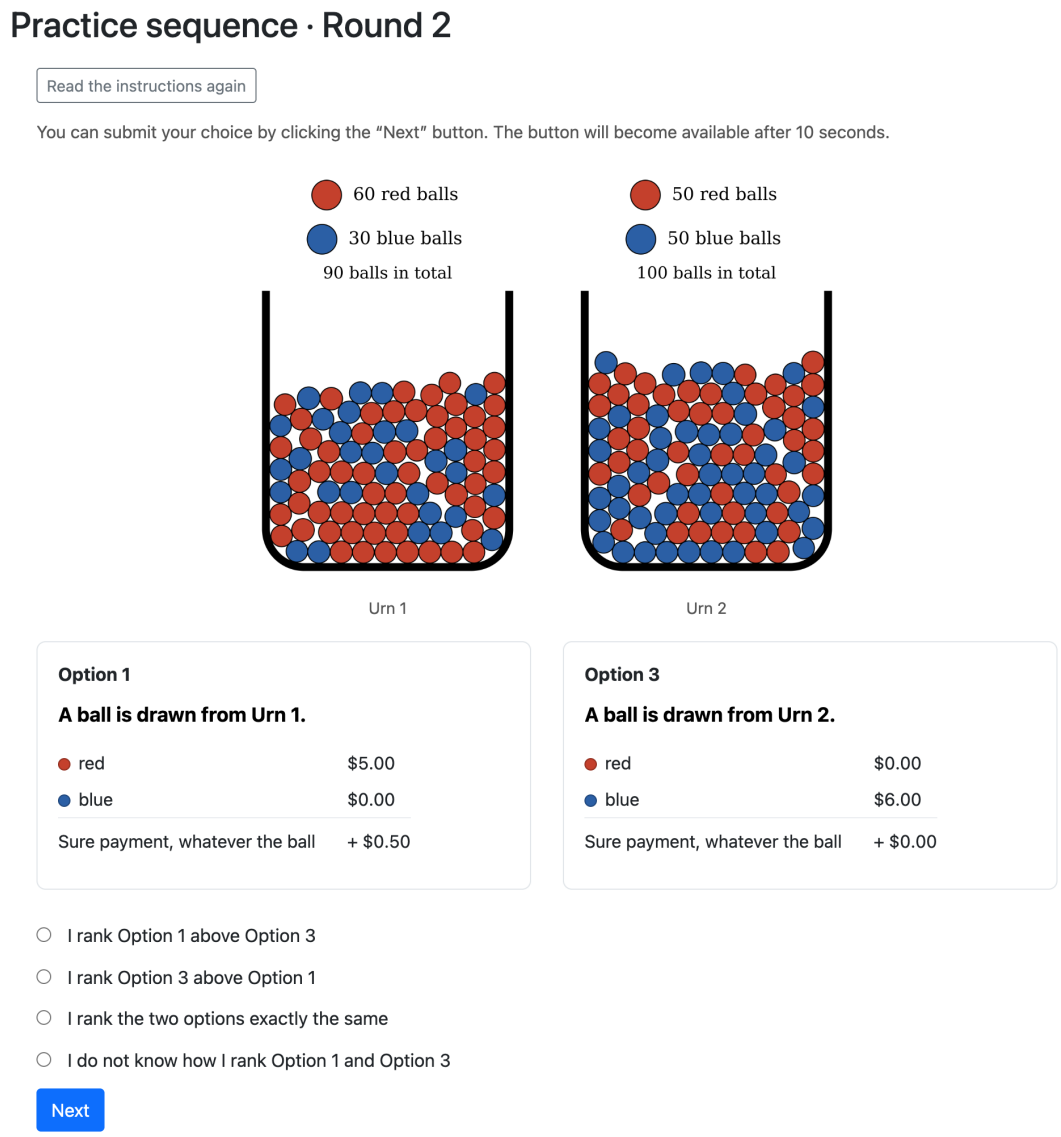
Answering honestly in each round will never harm you. Within each sequence, the option you hold at the end is not worse than any of the options you have seen. Because you do not know which options you will see in the future, make each choice according to your true preferences.

Next

Practice sequence, Round 1. After reading instructions, subjects work through a practice sequence of two rounds.



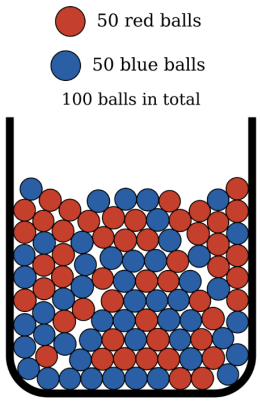
Practice sequence, Round 2. The option held after Round 1 is compared with a new option.



End of the practice sequence. The screen shows the option held at the end of the practice and explains what happens when a sequence ends. Subjects may start the practice over.

Practice sequence

The practice sequence has ended. The option you were holding at the end of this sequence:



Urn 2

A ball is drawn from Urn 2.

red	$0.00
blue	$6.00
Sure payment, whatever the ball	+ $0.00

The practice has only one sequence. In the real decisions, there may be several. Each time a sequence ends, one of two things happens:

- **If it was the last one**, the option you are holding is the one that determine your payoff: a ball is drawn from its urn and you are paid what that option gives.
- **If it was not the last one**, a new sequence begins and this option will not be carried on.

Start the practice over

Next

Comprehension quiz (1 of 3). Subjects must answer all six questions correctly to proceed, and receive a $2.00 bonus if they do so on the first attempt. Questions 1 and 2 cover the payment rule.

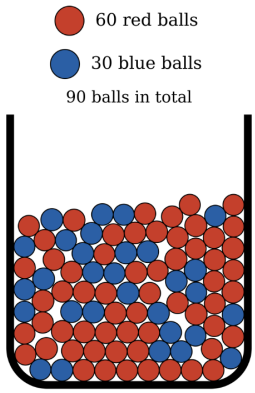
Comprehension Quiz

Read the instructions again

There are some questions to evaluate your comprehension. If you answer all the questions correctly **on the first attempt**, you will get **a bonus of $2.00**. You must answer the quiz correctly in order to proceed to the decision problems. You can read the instructions again if needed.

Q1. At the end, the money you actually receive is determined by:

- ○ A question fixed before you started.
- ○ The average of all your choices.
- ○ The single option you are holding when all the sequence(s) end.



Urn 1

Option X

A ball is drawn from Urn 1.

● red	$0.00
● blue	$8.00
Sure payment, whatever the ball	+ $0.25

Q2. Suppose the option above is the one you are holding when your choices end, and the ball drawn is red. You earn:

- ○ $0.25
- ○ $0.00
- ○ $8.25

Comprehension quiz (2 of 3). Questions 3 and 4 cover the reset between sequences and repeated comparisons.

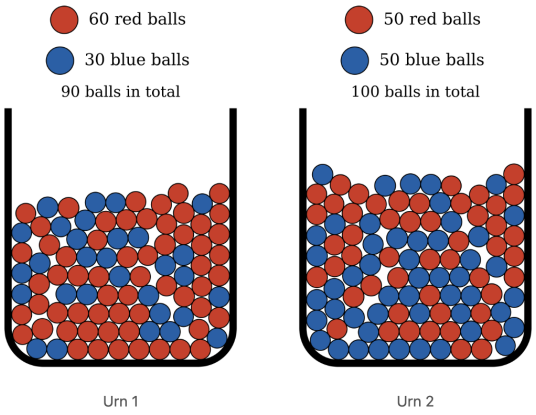
Q3. If one sequence ends and a new sequence begins. What happens to the option you were holding at the end of the earlier sequence?

- ○ It depends -- the computer decides at random whether it carries over.
- ○ It carries forward to the next sequence, and is compared with the first option of the new sequence.
- ○ It does not carry forward to the next sequence. The new sequence starts with two new options.

Q4. Suppose you compared two options and gave an answer. If the same two options appear again and you are asked again how you rank them:

- ○ You can change your earlier answer at a cost.
- ○ Your earlier answer is already recorded on the screen and cannot be changed.
- ○ You are asked to compare them again, and may answer differently.

Comprehension quiz (3 of 3). Question 5 covers the binding rule and carry-on rule after a report of "cannot rank".



Option X

A ball is drawn from Urn 1.

red	$5.00
blue	$0.00
Sure payment, whatever the ball	+ $0.50

Option Y

A ball is drawn from Urn 2.

red	$0.00
blue	$6.00
Sure payment, whatever the ball	+ $0.00

Q5. Suppose you compared the two options above and answered "I do not know how I rank Option X and Option Y". Later, the comparison below appears. They have the same two bets, but the first one now pays $0.70 for sure instead of $0.50. What happens?

Option Z

A ball is drawn from Urn 1.

red	$5.00
blue	$0.00
Sure payment, whatever the ball	+ $0.70

Option Y

A ball is drawn from Urn 2.

red	$0.00
blue	$6.00
Sure payment, whatever the ball	+ $0.00

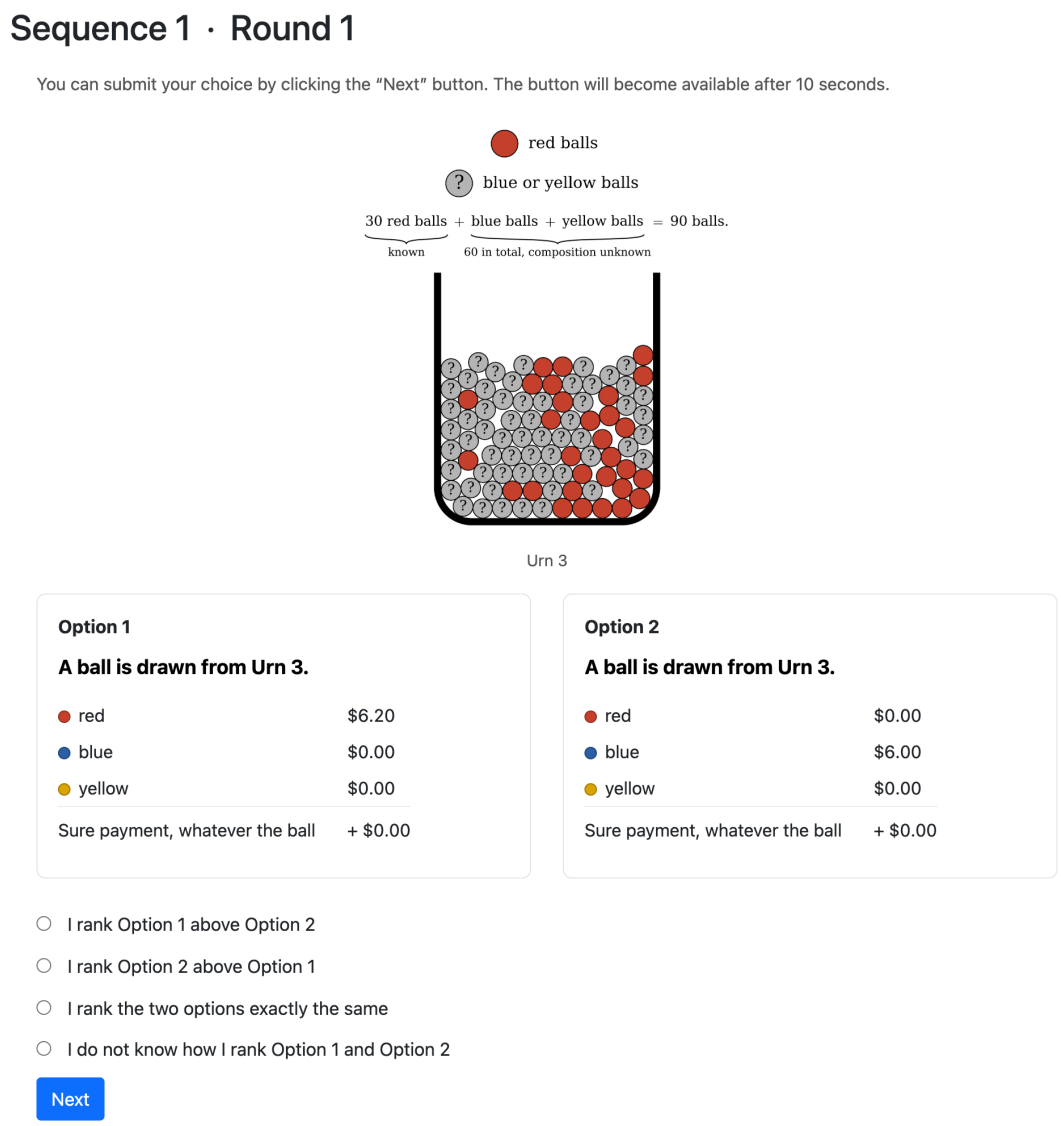
- ○ The option paying $0.70 for sure is automatically counted as the better one.
- ○ Your earlier answer is already recorded here: the sure payment changed by less than $0.25, so it still applies.
- ○ You are asked to rank them again, and can answer freely.

Q6. Suppose you answer "I do not know how I rank Option X and Option Y" between the option you hold and the new option. Which option do you hold afterwards?

- ○ The computer decides at random.
- ○ The new option.
- ○ The option I was already holding.

Next

Sequence 1, Round 1. Subjects see each list as a sequence of rounds. In these screenshots Sequence 1 is List 4. The subject holds Option 1 (A_1) and meets Option 2 (B_1).



Sequence 1, Round 2. The subject kept Option 1 (A_1) and meets Option 3 (A_2).

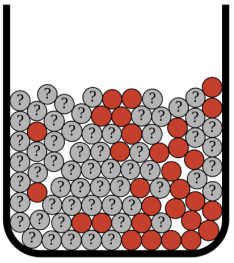
Sequence 1 · Round 2

You can submit your choice by clicking the "Next" button. The button will become available after 10 seconds.

red balls

? blue or yellow balls

30 red balls + blue balls + yellow balls = 90 balls.
known — 60 in total, composition unknown



Urn 3

Option 1

A ball is drawn from Urn 3.

red	$6.20
blue	$0.00
yellow	$0.00
Sure payment, whatever the ball	+ $0.00

Option 3

A ball is drawn from Urn 3.

red	$6.00
blue	$0.00
yellow	$6.00
Sure payment, whatever the ball	+ $0.00

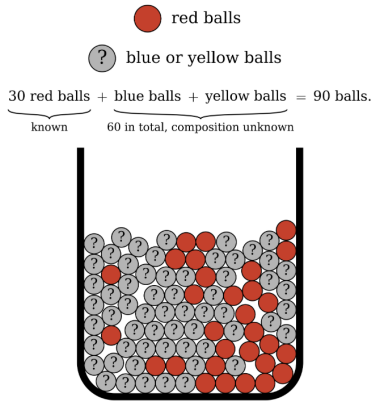
- ○ I rank Option 1 above Option 3
- ○ I rank Option 3 above Option 1
- ○ I rank the two options exactly the same
- ○ I do not know how I rank Option 1 and Option 3

Next

Sequence 1, Round 3. The subject now holds Option 3 (A_2) and meets Option 4 (B_2).

Sequence 1 · Round 3

You can submit your choice by clicking the "Next" button. The button will become available after 10 seconds.



Urn 3

Option 3

A ball is drawn from Urn 3.

red	$6.00
blue	$0.00
yellow	$6.00
Sure payment, whatever the ball	+ $0.00

Option 4

A ball is drawn from Urn 3.

red	$0.00
blue	$6.20
yellow	$6.20
Sure payment, whatever the ball	+ $0.00

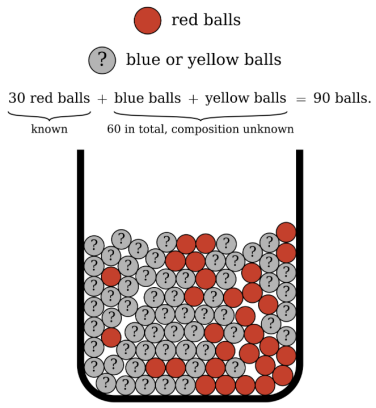
- ○ I rank Option 3 above Option 4
- ○ I rank Option 4 above Option 3
- ○ I rank the two options exactly the same
- ○ I do not know how I rank Option 3 and Option 4

Next

Sequence 1, Round 4. The subject holds Option 3 (A_2) and meets Option 5 (F), the dominated option at the end of List 4.

Sequence 1 · Round 4

You can submit your choice by clicking the "Next" button. The button will become available after 10 seconds.



Urn 3

Option 3

A ball is drawn from Urn 3.

red	$6.00
blue	$0.00
yellow	$6.00
Sure payment, whatever the ball	+ $0.00

Option 5

A ball is drawn from Urn 3.

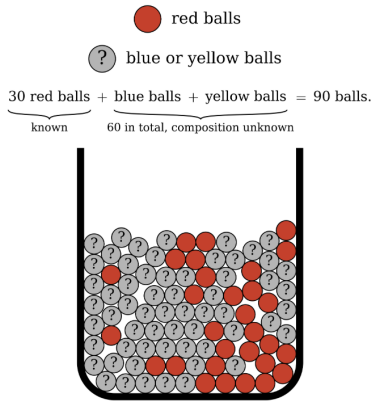
red	$0.00
blue	$0.00
yellow	$2.00
Sure payment, whatever the ball	+ $0.00

- ○ I rank Option 3 above Option 5
- ○ I rank Option 5 above Option 3
- ○ I rank the two options exactly the same
- ○ I do not know how I rank Option 3 and Option 5

Next

End of Sequence 1. The screen shows the option held at the end of the sequence and announces that a new sequence will begin.

This sequence has ended. The option you were holding at the end of this sequence:



Urn 3

The option you were holding

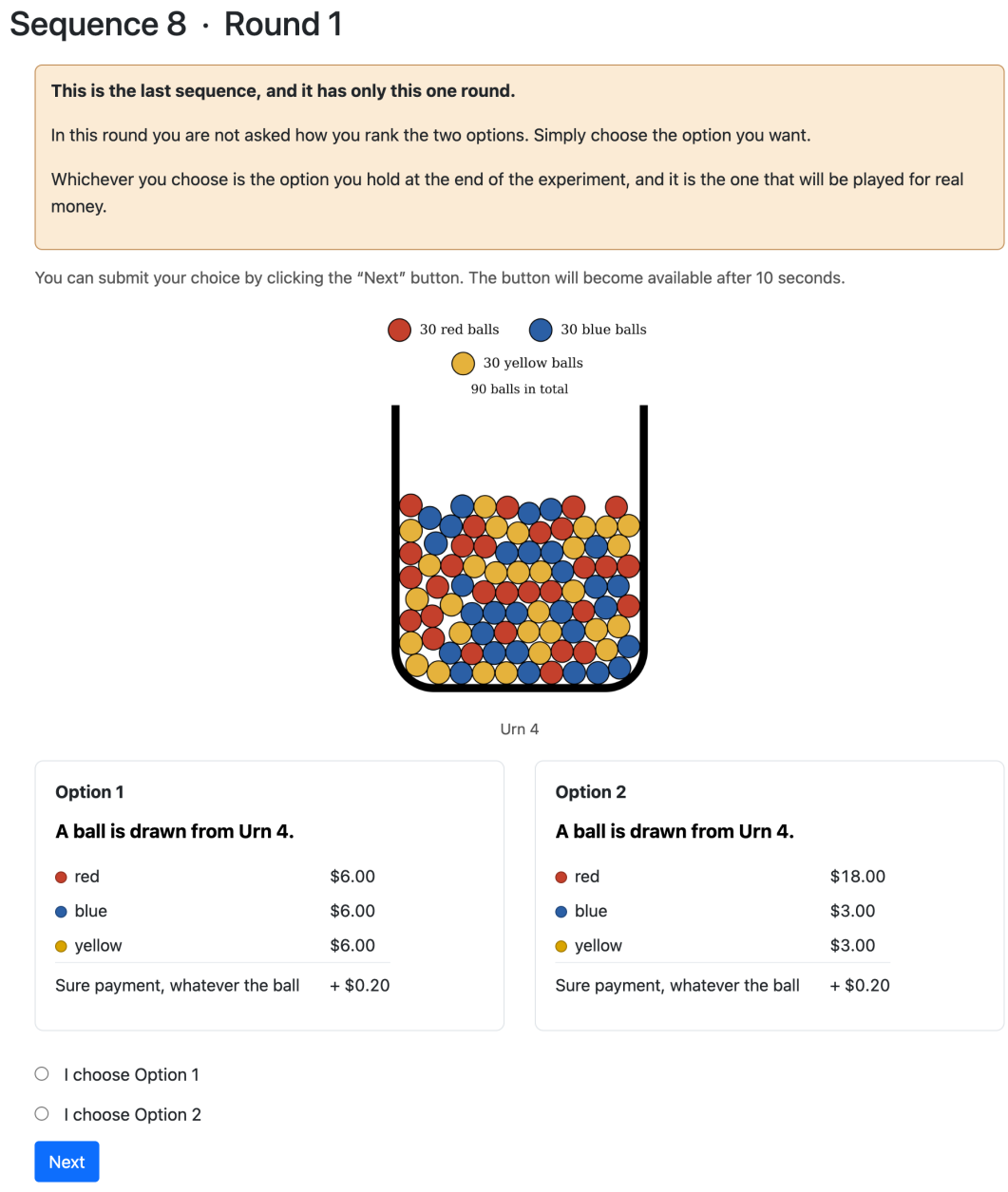
A ball is drawn from Urn 3.

red	$0.00
blue	$0.00
yellow	$2.00
Sure payment, whatever the ball	+ $0.00

A new sequence will begin. Click "next" to proceed.

Next

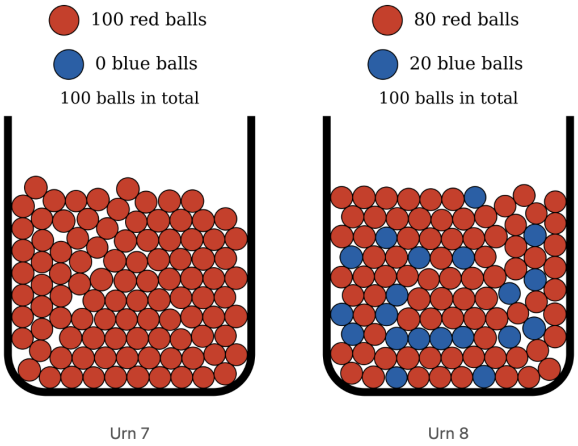
Verification round. Shown only in the verification arms, after the 7-th list. It repeats a comparison the subject has already answered, here Z_2 against Y, and asks for a straight choice. The option chosen determines the payment.



Survey, Part 1. After the final list, subjects describe in their own words how they thought about a comparison they made earlier.

Survey: Part 1

You have now finished the choice tasks. We would like to better understand how people think about these decisions. We appreciate the time and care you take in answering. Please consider the following choice again:



Option X	
A ball is drawn from Urn 7.	
red	$6.00
blue	$0.00
Sure payment, whatever the ball	+ $0.00

Option Y	
A ball is drawn from Urn 8.	
red	$8.20
blue	$0.00
Sure payment, whatever the ball	+ $0.00

How would you describe your thoughts about this choice? There is no right or wrong way to think about it. You might have a clear preference, consider the two options equally good, find it difficult to decide, or think about it in some other way. You may also describe how you approached the comparison, or how you arrived at your decision. Please describe your thinking in your own words. (at least 50 characters).

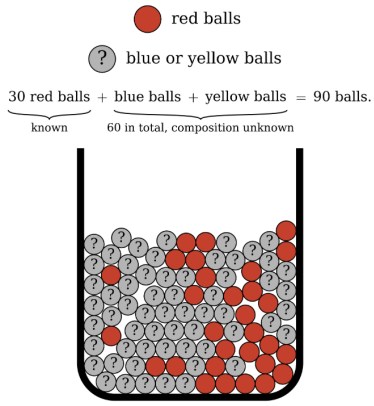
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Next

Survey, Part 2. The same question for a second comparison.

Survey: Part 2

You have now finished the choice tasks. We would like to better understand how people think about these decisions. We appreciate the time and care you take in answering. Please consider the following choice again:



Urn 3

Option X		Option Y	
A ball is drawn from Urn 3.		**A ball is drawn from Urn 3.**	
red	$6.20	red	$0.00
blue	$0.00	blue	$6.00
yellow	$0.00	yellow	$0.00
Sure payment, whatever the ball	+ $0.00	Sure payment, whatever the ball	+ $0.00

How would you describe your thoughts about this choice? There is no right or wrong way to think about it. You might have a clear preference, consider the two options equally good, find it difficult to decide, or think about it in some other way. You may also describe how you approached the comparison, or how you arrived at your decision. Please describe your thinking in your own words. (at least 50 characters).

0 characters

Next

B.2 Multi-list AMC-2

Instructions (1 of 3).

Instructions

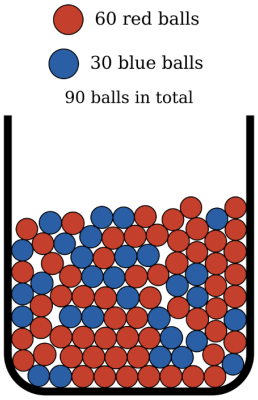
Please read the instructions carefully. The Next button will become available after 3 minutes.

Introduction

In this experiment you will make a series of choices between two options. At the end, one option will be played for real money.

What an option is

Each option is a **bet on the colour of a ball drawn from an urn**, together with a **sure payment** you receive no matter which ball is drawn. The urn's contents are always shown on your screen. For example:



Urn 1

Option X

A ball is drawn from Urn 1.

red	$5.00
blue	$0.00
Sure payment, whatever the ball	+ $0.50

If the ball drawn from Urn 1 is red you receive $5.00 from the bet, otherwise the bet pays nothing, and either way you also receive $0.50 for sure.

Instructions (2 of 3).

Making choices

Your choices are organized into **sequences** of rounds. Within each sequence, options are numbered in the order in which they appear: Option 1, Option 2, Option 3, and so on.

At the beginning of each sequence, you start by holding **Option 1** and compare it with **Option 2**. In each later round, the option you currently hold is compared with a new option.

For example, in a round you see Option X and Option Y side by side, and you are asked to choose one of:

- "I rank Option X above Option Y"
- "I rank Option Y above Option X"
- "I rank the two options exactly the same"
- "I do not know how I rank Option X and Option Y"

There are no right or wrong answers. Choose the answer that best describes your preference.

How your choice determines what you hold

On each screen, the option you currently hold is shown on the left, and a new option is shown on the right.

For example, suppose you currently hold Option X, and the new option is Option Y:

- If you choose "I rank X above Y", you continue to hold X.
- If you choose "I rank Y above X", you replace X with Y.
- If you choose "I rank the two options exactly the same", you replace X with Y.
- If you choose "I do not know how I rank X and Y" the experiment checks Y against any options that were previously kept aside. A kept-aside option is an option that was previously involved in an "I do not know how I rank them" answer but did not become your current option.
  - If you rank at least one kept-aside option above Y, you continue to hold X, and Y is kept aside for a possible later comparison.
  - If you rank none of the kept-aside options above Y, Y becomes your new current option, and X is kept aside for a possible later comparison.
  - If there are no options kept aside, Y becomes your new current option, and X is kept aside for a possible later comparison.

Keeping some options aside allows the experiment to check a new option against options that you previously could not rank. Any option you saw previously but did not keep aside has already been ruled out by your previous choices, so it does not need to be considered again. **Together, these rules ensure that the option you hold after each round is not worse than any of the options you have seen so far.**

The option you hold becomes your current option in the next round. It is compared with the next new option, and the same process continues until the sequence ends. You do not need to keep track of which options have been kept aside. The experiment will keep track of them for you.

Sequence structure

The experiment is divided into sequences. When a new sequence begins, the option you were holding and all options kept aside from the previous sequence are reset. You start the new sequence by holding **Option 1** and comparing it with **Option 2**. You continue this process until the end.

What you know and do not know

At the start of the experiment, you are not told how many sequences there are or how many rounds each sequence contains. Therefore, the experiment may end after any round.

In addition, you will not know which options are still to come. You will only see the options that you have already encountered and the two options shown on your current screen. The set of options is determined in advance by the experimenter, and you will encounter all of the options in the experiment.

Instructions (3 of 3).

Repeated comparisons

An option can appear more than once, so a comparison you answer early may appear again at any later point in the experiment. If you are again asked how you rank two options, and you have already answered for that pair, you will see your earlier answer already recorded and you cannot change it, even if the options appear in a different order. Therefore, answer each comparison according to your true preference.

Comparisons with small payment changes

If you choose "I do not know how I rank the two options" for a pair of bets, your answer will also apply to a later comparison involving the same two bets if the sure payment attached to one of the options has changed by less than $0.25.

For example, suppose you answer "I do not know how I rank the two options" for:

- Option A: a bet on red from urn 1, prize $6, plus $0.00 for sure
- Option B: a bet on blue from urn 1, prize $6, plus $0.00 for sure

Later, you may see the same two bets, but Option A now offers $0.10 for sure:

- Option A: a bet on red from urn 1, prize $6, plus **$0.10** for sure
- Option B: a bet on blue from urn 1, prize $6, plus $0.00 for sure

Because the sure payment changed by $0.10, which is less than $0.25, if you are asked to rank the two options, your previous "I do not know how I rank the two options" answer will already be recorded and you cannot change the answer.

This rule applies only when your original answer was "I do not know how I rank Option X and Option Y". **In other words, by saying you do not know how you rank two options, you are also saying that adding or removing a sure payment of less than $0.25 to one of them would not settle the matter for you. Answering truthfully that you do not know how you rank two options will not hurt you.**

However, if you do know how you rank two options and still choose that answer, it may hurt you. For example, suppose the two options in a round are:

- Option A: a bet on red from an urn with 50 red and 50 blue balls, prize $6, plus $0.00 for sure
- Option B: a bet on blue from the same urn, prize $6, plus $0.00 for sure

Most people rank these two options exactly the same. Suppose you nevertheless answer "I do not know how I rank Option A and Option B". Later, you may see the same two options again, but Option A now offers $0.10 for sure:

- Option A: a bet on red from the same urn, prize $6, plus **$0.10** for sure
- Option B: a bet on blue from the same urn, prize $6, plus $0.00 for sure

If you are asked to rank the two options, because of your earlier answer this comparison is already answered for you and you cannot change it, even though Option A is now clearly better than Option B.

Payment

Whichever option you are holding when the experiment ends is your payoff. A ball is then drawn from that option's urn, and you are paid the prize for the colour drawn plus that option's sure payment.

Your best strategy

Answering honestly in each round will never harm you. Within each sequence, the option you hold at the end is not worse than any of the options you have seen. Because you do not know which options you will see in the future, make each choice according to your true preferences.

Next

Comprehension quiz (1 of 3). Questions 1 to 5 are the same as in *Multi-list AMC-1*, with the answers in a different order.

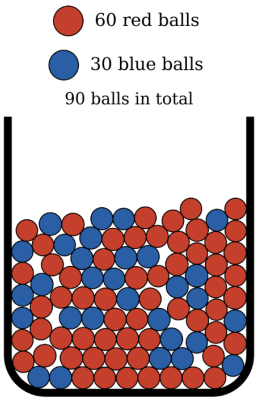
Comprehension Quiz

Read the instructions again

There are some questions to evaluate your comprehension. If you answer all the questions correctly **on the first attempt**, you will get **a bonus of $2.00**. You must answer the quiz correctly in order to proceed to the decision problems. You can read the instructions again if needed.

Q1. At the end, the money you actually receive is determined by:

- ○ The average of all your choices.
- ○ The single option you are holding when all the sequence(s) end.
- ○ A question fixed before you started.



Urn 1

Option X

A ball is drawn from Urn 1.

red	$0.00
blue	$8.00
Sure payment, whatever the ball	+ $0.25

Q2. Suppose the option above is the one you are holding when your choices end, and the ball drawn is red. You earn:

- ○ $8.25
- ○ $0.25
- ○ $0.00

Comprehension quiz (2 of 3). Questions 3 and 4.

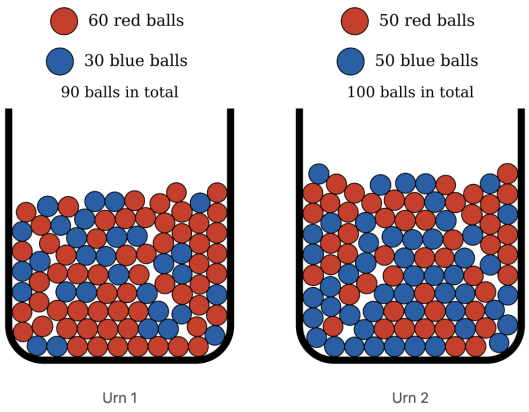
Q3. If one sequence ends and a new sequence begins. What happens to the option you were holding at the end of the earlier sequence?

- ○ It depends -- the computer decides at random whether it carries over.
- ○ It does not carry forward to the next sequence. The new sequence starts with two new options.
- ○ It carries forward to the next sequence, and is compared with the first option of the new sequence.

Q4. Suppose you compared two options and gave an answer. If the same two options appear again and you are asked again how you rank them:

- ○ Your earlier answer is already recorded on the screen and cannot be changed.
- ○ You are asked to compare them again, and may answer differently.
- ○ You can change your earlier answer at a cost.

Comprehension quiz (3 of 3). Question 6 asks what happens after the answer "I do not know how I rank" under *Multi-list AMC-2*.



Option X	
A ball is drawn from Urn 1.	
red	$5.00
blue	$0.00
Sure payment, whatever the ball	+ $0.50

Option Y	
A ball is drawn from Urn 2.	
red	$0.00
blue	$6.00
Sure payment, whatever the ball	+ $0.00

Q5. Suppose you compared the two options above and answered "I do not know how I rank Option X and Option Y". Later, the comparison below appears. They have the same two bets, but the first one now pays $0.70 for sure instead of $0.50. What happens?

Option Z	
A ball is drawn from Urn 1.	
red	$5.00
blue	$0.00
Sure payment, whatever the ball	+ $0.70

Option Y	
A ball is drawn from Urn 2.	
red	$0.00
blue	$6.00
Sure payment, whatever the ball	+ $0.00

- ○ The option paying $0.70 for sure is automatically counted as the better one.
- ○ You are asked to rank them again, and can answer freely.
- ○ Your earlier answer is already recorded here: the sure payment changed by less than $0.25, so it still applies.

Q6. Suppose you answer "I do not know how I rank Option X and Option Y" between the option you hold and the new option. What happens next?

- ○ I always hold my current option, no matter what.
- ○ I always hold the new option.
- ○ The new option is compared with options kept aside earlier; whether I hold my current option or the new one depends on those comparisons.

Next

B.3 Multi-list ABC

Instructions (1 of 2).

Instructions

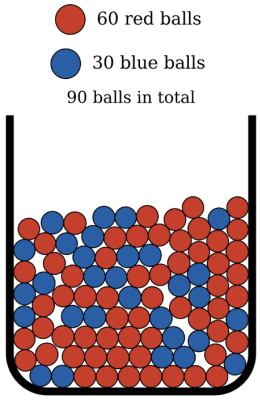
Please read the instructions carefully. The Next button will become available after 3 minutes.

Introduction

In this experiment you will make some choices between two options. Your choices determine your payoff.

What an option is

Each option is a **bet on the colour of a ball drawn from an urn**, together with a **sure payment** you receive no matter which ball is drawn. The urn's contents are always shown on your screen. For example:



Urn 1

Option X

A ball is drawn from Urn 1.

red	$5.00
blue	$0.00
Sure payment, whatever the ball	+ $0.50

If the ball drawn from Urn 1 is red you receive $5.00 from the bet, otherwise the bet pays nothing, and either way you also receive $0.50 for sure.

Instructions (2 of 2).

Making choices

Your choices are organized into **sequences** of rounds. Within each sequence, options are numbered in the order in which they appear: Option 1, Option 2, Option 3, and so on.

At the beginning of each sequence, you start by holding **Option 1** and compare it with **Option 2**. In each later round, the option you currently hold is compared with a new option.

For example, in a round you see Option X and Option Y side by side, and you are asked to choose one of:

- "I rank Option X above Option Y"
- "I rank Option Y above Option X"

There are no right or wrong answers. Choose the answer that best describes your preference. **If you rank the two options exactly the same, either answer is correct.**

How your choice determines what you hold

On each screen, the option you currently hold is shown on the left, and a new option is shown on the right.

For example, suppose you currently hold Option X, and the new option is Option Y:

- If you choose "I rank X above Y", you continue to hold X.
- If you choose "I rank Y above X", you replace X with Y.

The option you hold becomes your current option in the next round. It is compared with the next new option, and the same process continues until the sequence ends.

Sequence structure

The experiment is divided into sequences. When a new sequence begins, the option you were holding is reset. You start the new sequence by holding **Option 1** and comparing it with **Option 2**. You continue this process until the end.

What you know and do not know

At the start of the experiment, you are not told how many sequences there are or how many rounds each sequence contains. Therefore, the experiment may end after any round.

In addition, you will not know which options are still to come. You will only see the options that you have already encountered and the two options shown on your current screen. The set of options is determined in advance by the experimenter, and you will encounter all of the options in the experiment.

Repeated comparisons

An option can appear more than once, so a comparison you answer early may appear again at any later point in the experiment. If you are again asked how you rank two options, and you have already answered for that pair, you will see your earlier answer already recorded and you cannot change it, even if the options appear in a different order. Therefore, answer each comparison according to your true preference.

Payoff

Whichever option you are holding when the experiment ends is your payoff. A ball is then drawn from that option's urn, and you are paid the prize for the colour drawn plus that option's sure payment.

Your best strategy

Answering honestly in each round will never harm you. Within each sequence, the option you hold at the end is not worse than any of the options you have seen. Because you do not know which options you will see in the future, make each choice according to your true preferences.

Next

Comprehension quiz (1 of 2). The quiz has four questions. Questions 1 and 2 cover the payment rule.

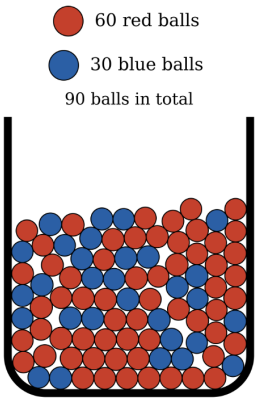
Comprehension Quiz

Read the instructions again

There are some questions to evaluate your comprehension. If you answer all the questions correctly **on the first attempt**, you will get **a bonus of $2.00**. You must answer the quiz correctly in order to proceed to the decision problems. You can read the instructions again if needed.

Q1. At the end, the money you actually receive is determined by:

- ○ A question fixed before you started.
- ○ The single option you are holding when all the sequence(s) end.
- ○ The average of all your choices.



Urn 1

Option X

A ball is drawn from Urn 1.

● red	$0.00
● blue	$8.00
Sure payment, whatever the ball	+ $0.25

Q2. Suppose the option above is the one you are holding when your choices end, and the ball drawn is red. You earn:

- ○ $0.25
- ○ $0.00
- ○ $8.25

Comprehension quiz (2 of 2). Questions 3 and 4 cover the reset between sequences and repeated comparisons.

Q3. If one sequence ends and a new sequence begins. What happens to the option you were holding at the end of the earlier sequence?

- ○ It carries forward to the next sequence, and is compared with the first option of the new sequence.
- ○ It depends -- the computer decides at random whether it carries over.
- ○ It does not carry forward to the next sequence. The new sequence starts with two new options.

Q4. Suppose you compared two options and gave an answer. If the same two options appear again and you are asked again how you rank them:

- ○ Your earlier answer is already recorded on the screen and cannot be changed.
- ○ You are asked to compare them again, and may answer differently.
- ○ You can change your earlier answer at a cost.

Next